Is a maximally entangled state maximally entangled?

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Outline

- Entanglement
- How to quantify entanglement
- Different measures
- Conclusion

EPR & Bell

- EPR believed in local realism
- Translates into a theory with local hidden variables (LHV)
- Bell showed that such a theory is impossible by bounding the value of a measurement operator of such a theory and by showing that QM can violate this bound

Entanglement

- Product state
  - $\rho = \rho_A \otimes \rho_B$
- Entangled state
  - Not a product state
- Multipartite generalization
  - Trivial

How to quantify entanglement and non-locality

- Open question
- Subject to debate
- Many measures have been to put forward

Communication

Entangled State ($|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$)

LHVs

$x \in M_A$

$y \in M_B$

Alice

Classical Communication

Bob

a

b

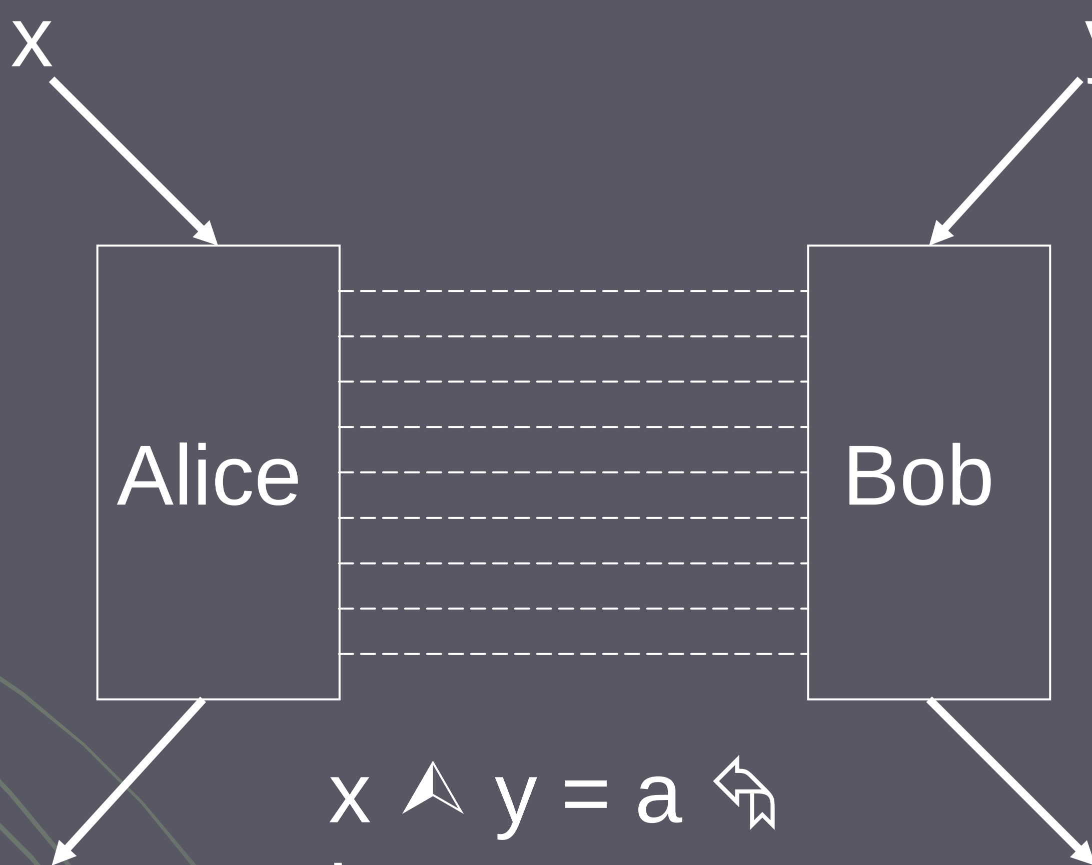
$Pr_{sim}[a,b|x,y] = Pr_{quantum}[a,b|x,y]$

$Pr_{quantum}[a,b|x,y]$

Communication

- For a Bell state $|\phi+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$: 1 bit is sufficient in the worst case
- For $|\phi\rangle = \alpha|00\rangle + \beta|11\rangle$, where $\alpha,\beta \neq 1/\sqrt{2}, 0$: 2 bits are necessary in the worst case
- ☹ Discrepancy also exist in the expected communication scenario

Non-local boxes

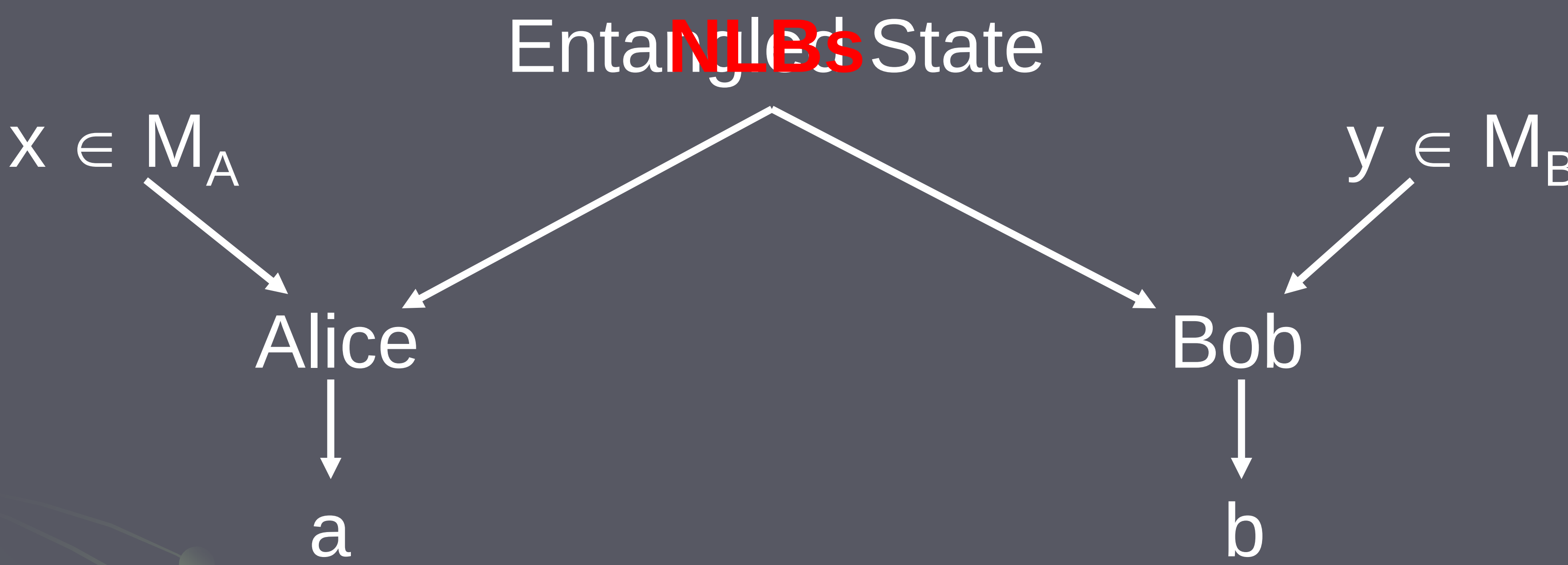


$x \wedge y = a \oplus b$

$a \in_r \{0,1\}$

$b \in_r \{0,1\}$

Non-local boxes



$Pr_{sim}[a,b|x,y] = Pr_{quantum}[a,b|x,y]$

$Pr_{quantum}[a,b|x,y]$

Non-local Boxes

- For a Bell state $|\phi+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$: 1 NLB is sufficient in the worst case
- For $|\phi\rangle = \alpha|00\rangle + \beta|11\rangle$, where $\alpha,\beta \neq 1/\sqrt{2}, 0$: 2 NLBs are necessary in the worst case

Detection loop-hole

Entangled State

LHVs

$x \in M_A$ $y \in M_B$

Alice Bob

a or ϒ b or ϒ

$Pr_{sim}[a,b|x,y] = Pr_{quantum}[a,b|x,y]$

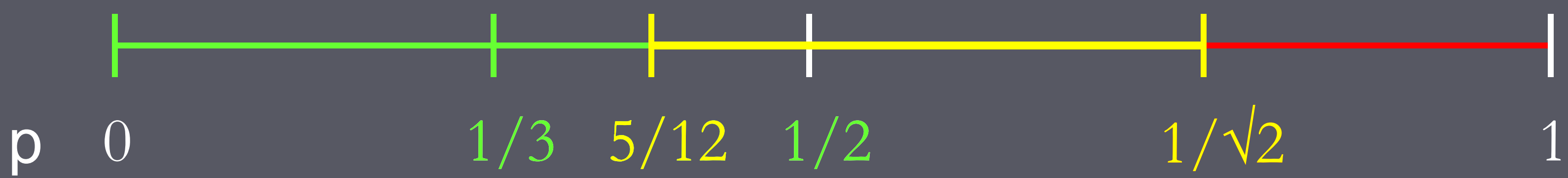
Max prob of ϒ

Detection loop-hole

- For a Bell state $|\phi+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$: 25%
- For $|\phi\rangle = \alpha|00\rangle + \beta|11\rangle$, where $\alpha,\beta \neq 1/\sqrt{2}, 0$: 33,3%

White Noise

$$p\,|\psi^-\rangle\langle\psi^-| + (1-p)\mathbb{1}/4$$



- $p \leq 1/3$: state is separable
- $p \leq 1/2$: can simulate vN on state with LHVs
- $p \leq 5/12$: can simulate POVMs on state with LHVs
- $p \geq 1/\sqrt{2}$: can't simulate with only LHVs
- $5/12 \leq p \leq 1/\sqrt{2}$: what happens exactly?

Bell inequalities

$P(A_1=B_1) + P(B_1=A_2+1) + P(A_2=B_2) + P(B_2=A_1) - P(A_1=B_1-1) - P(B_1=A_2) - P(A_2=B_2-1) - P(B_2=A_1-1)$

- Classically ≤ 2
- Maximally entangled state of two qutrits = $4(2\sqrt{3}+3)/9 > 2$
- Non-maximally entangled state of two qutrits = $1+\sqrt{(11/3)} > 4(2\sqrt{3}+3)/9$

Bell inequalities without probabilities

- A Bell inequality without probabilities is a set of multipartite measurements on an entangled state where any local classical model, which is to attempt to simulate the probability distribution of the outputs given by quantum mechanics, will attribute a non zero probability to a measurement outcome that is forbidden by quantum mechanics or will never produce certain outcomes which are predicted with a non zero probability in quantum mechanics.

Bell inequalities without probabilities

- $|\Gamma\rangle = (|01\rangle + |10\rangle + |11\rangle)/\sqrt{3}$

Alice/Bob	00	01	10	11
$I \otimes I$	0	1/3	1/3	1/3
$H \otimes I$	1/6	2/3	1/6	0
$I \otimes H$	1/6	1/6	2/3	0
$H \otimes H$	3/4	1/12	1/12	1/12

Bell inequalities without probabilities

- Works for almost any random state except
  - Product states
  - ☠ Bell states

Kullback-Leibler distance

- The average amount of support in favor of Q against C per trial when the data are generated by Q is the so-called relative entropy or Kullback-Leibler divergence
- $D(q,c) = \sum_z p_q(z) \log(p_q(z)/p_c(z))$
- Maximally entangled state of two qutrits : 0.058
- Non-maximally entangled state of two qutrits : 0.077

Entanglement of formation

- Def: How many pairs of maximally entangled states are needed to create the state we want (LOCC)
- For non-maximally entangled pair of qubits?

☺ 1!

- Teleportation!

Pseudo-Telepathy

$$\Pr_{quantum}[(a,b,x,y) \in W] = 1$$

$$\Pr_{LHVs}[(a,b,x,y) \in W] < 1$$

Pseudo-telepathy

- ☺ So far, the only pseudo-telepathy games known are on maximally entangled states
- As not been proved that it is impossible on non-maxiamally entangled states
- As not been proved that non-maximally entangled states cannot lower the classical probability of winning

Why are there anomalies in all these measures?

?

Maybe…

- If I think of something

One thing I should tell you

- All these measures are related
  - I could give a whole other talk as to how they are related
    - If you can read french, I can send you my phd thesis… once it's finished
- Insights into one will lead to insights in others

Conclusion

- ☹ I don’t know anything!
- 😐 I really would love to discuss this
  - 😐 I think that understanding this would yield great insights into physics and into the power of entanglement as a computational resource
- ☺ If you want to know what I do when I am serious… check my website ☺!!