Quantum Chaos and Observation

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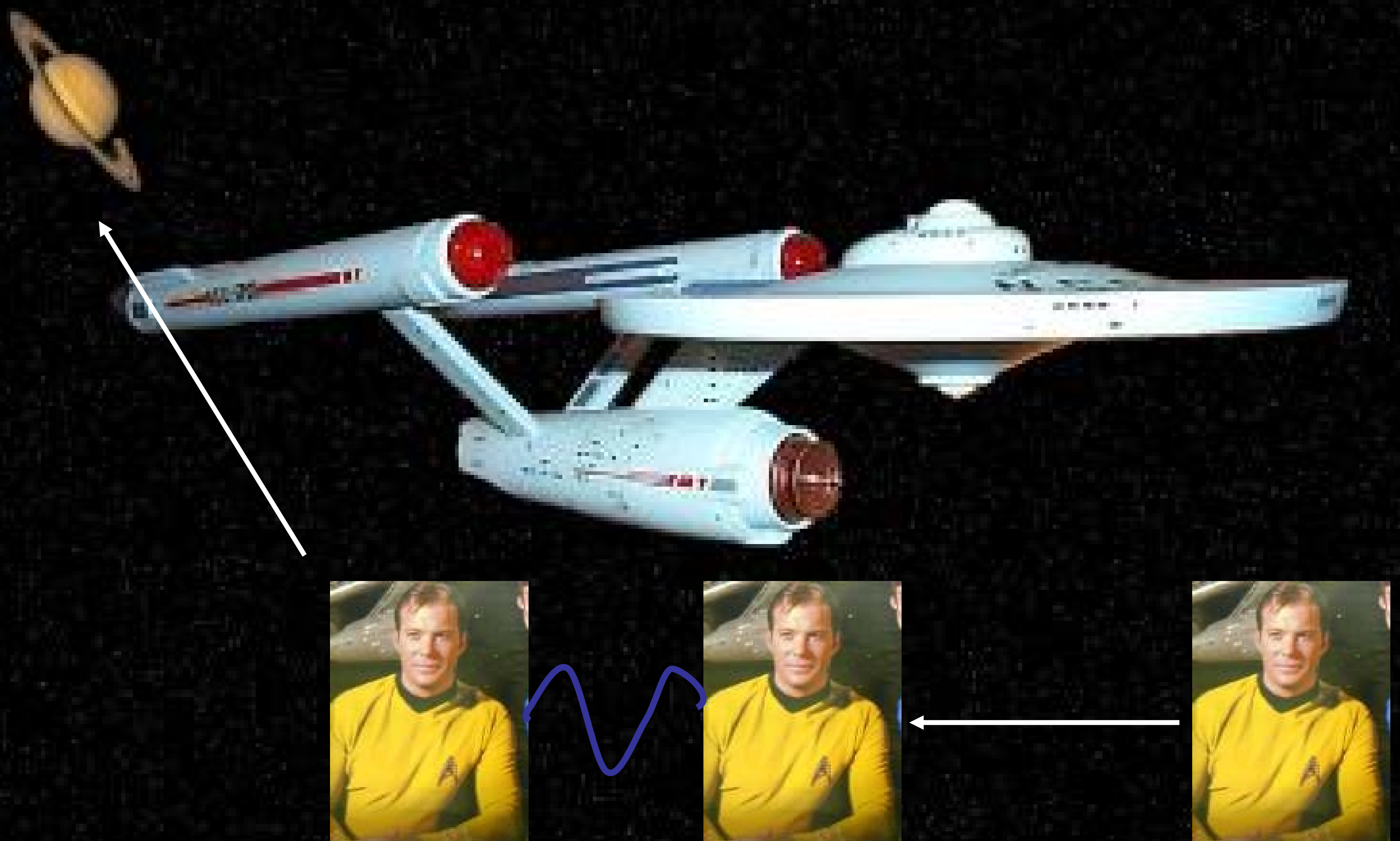
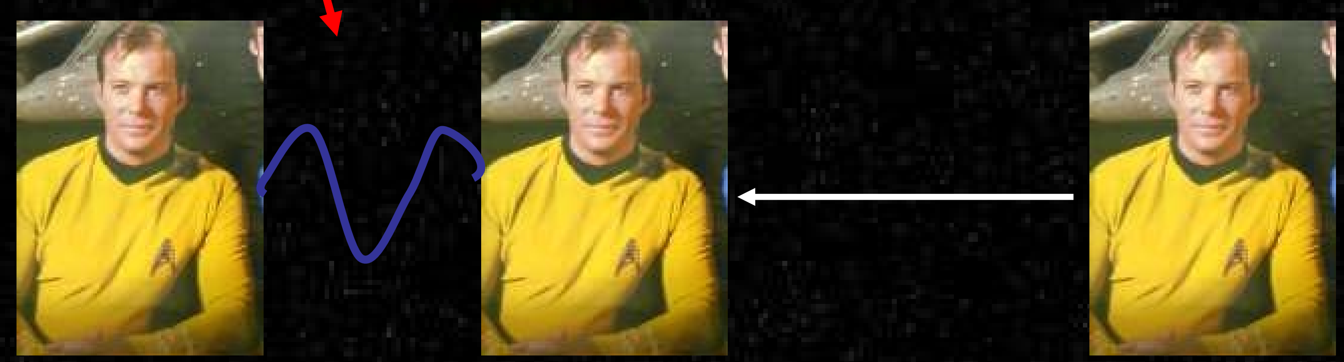
Big Question: Can you teleport big objects?



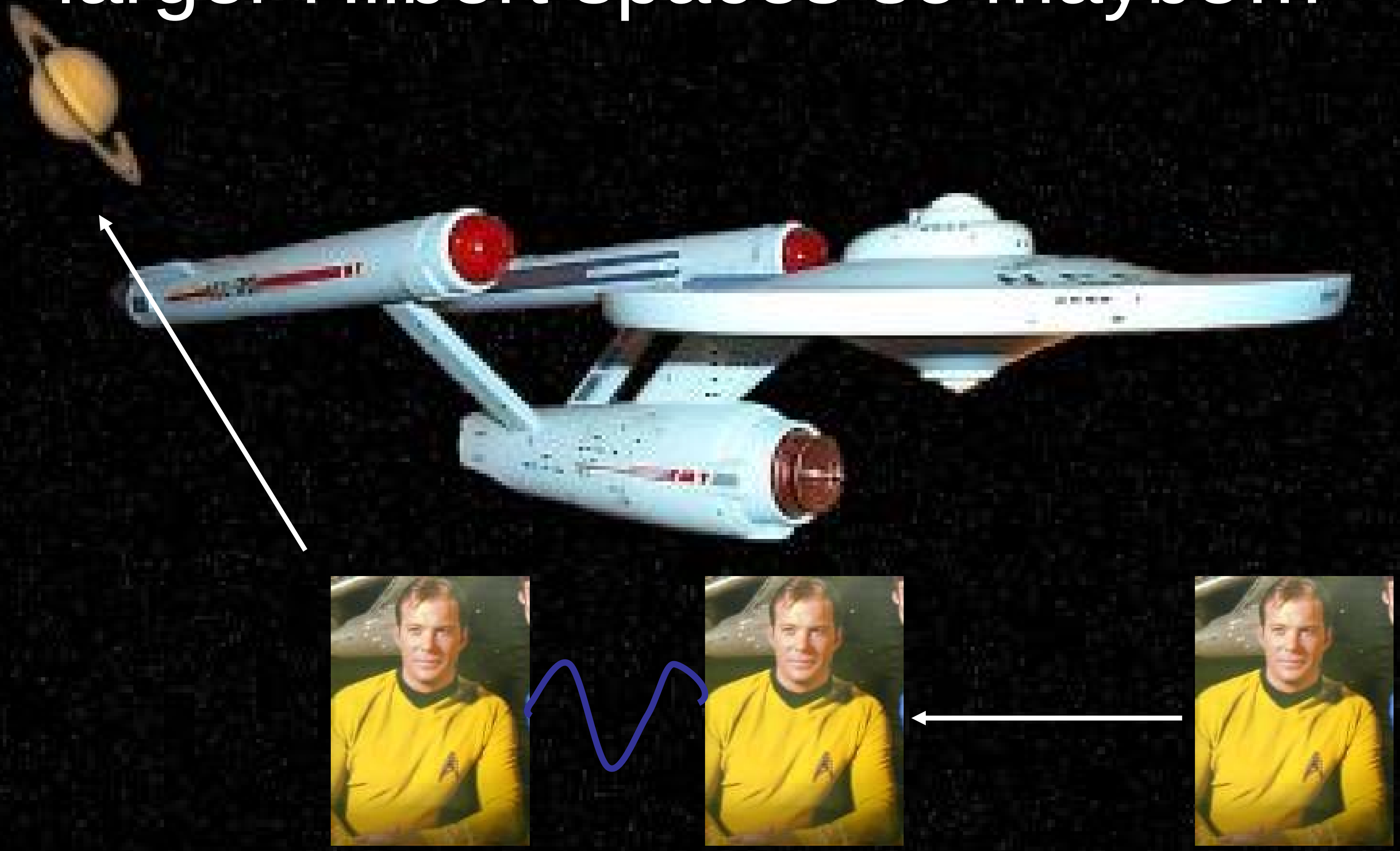
Problem: You need highly entangled states of dimension

$$2^{2^{6x10^{23}}}$$

Can you construct these states without a quantum computer that operates in a Hilbert space of this dimension?



Quantum Chaotic Operators can produce more entanglement for larger Hilbert spaces so maybe…



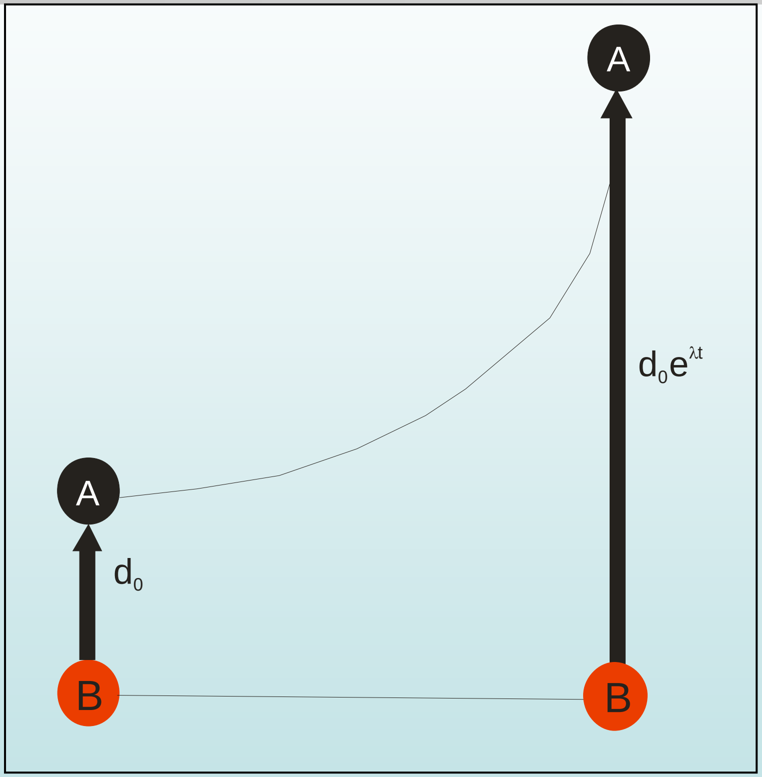

TILT-A-WHIRL
Now For A Limited time…
Decoherence Free!

Outline

- Classical Chaos
- Quantum Signatures of Chaos
- Measurement and Chaos
- Entanglement generation + Measurement
- Teleportation Fidelity for Qu-trits

Classical Chaos

- Discovered by Poincare in 1890 by studying the stability of the solar system.
- Chaos occurs regularly in fluid dynamics, plasma physics, and cellestial mechanics.
- Distance between adjacent trajectories increases exponentially like $d_0e^{-\lambda t}$. (λ is the lyapunov exponent)
- Classical chaos is technically deterministic, but is not predictable in a practical sense.



Phase Space Distributions

- Individual trajectories separate very quickly, however ensembles of trajectories do not.
- Phase space dist ρ gives the probability of finding a particle in (x,x+dx),(p,p+dp)
- Time evolution of probability distributions is given by the Liouville equation:

$$\frac{\partial\rho(x,p)}{\partial t}=\{H,\rho\}=-\frac{\partial\rho}{\partial x}\frac{\partial H}{\partial p}+\frac{\partial\rho}{\partial p}\frac{\partial H}{\partial x}$$

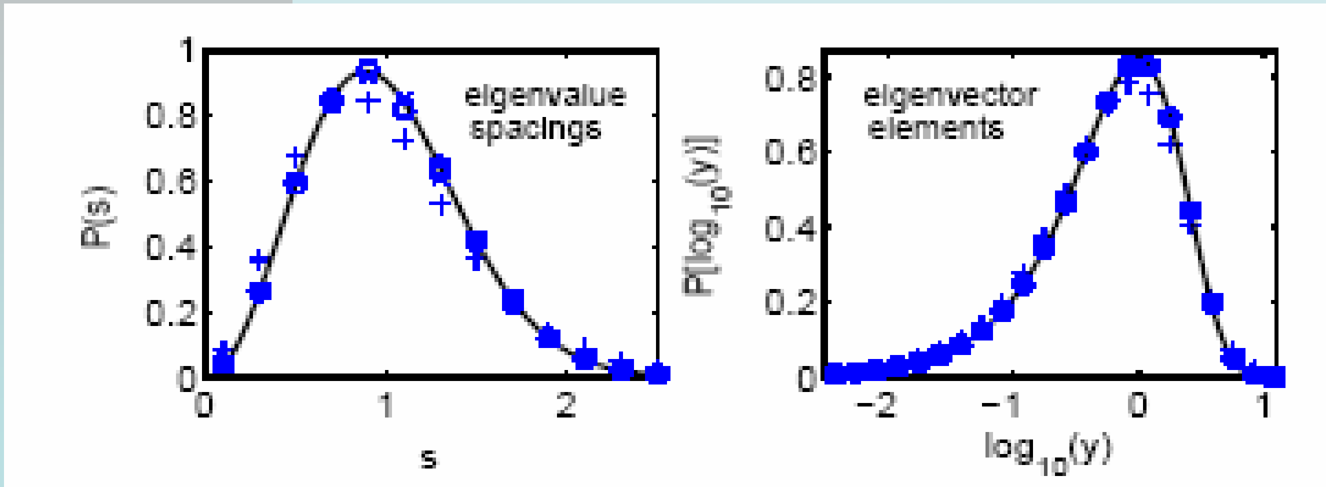
- This equation is linear, and thus exponential seperation will not occur for distributions.
- Area is also preserved under time evolution.

Quantum Chaos

- Quantum Chaos is simply the quantum mechanics of Hamiltonians that are classically chaotic.
- Lyapunov exponents are not a defining characteristic of Quantum Chaos.
- The classical limit of Quantum chaos as a result has been debated by Zurek et al.
- The classical limit for quantum chaotic systems typically happens from either decoherence or poor measurement.

Chaotic Generation Of Random Matrices

- Chaotic Hamiltonians generally have random level spacing, and eigenvectors.
- Fidelity with random matrix theory increases with dimension of Hilbert space.



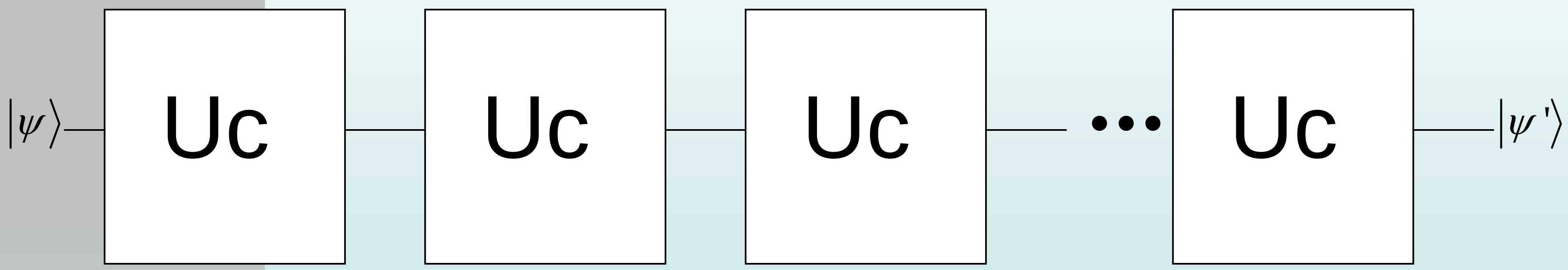
Yaakov S. Weinstein, and C. Stephen Hellberg

arXiv:quant-ph/0507103

Random States

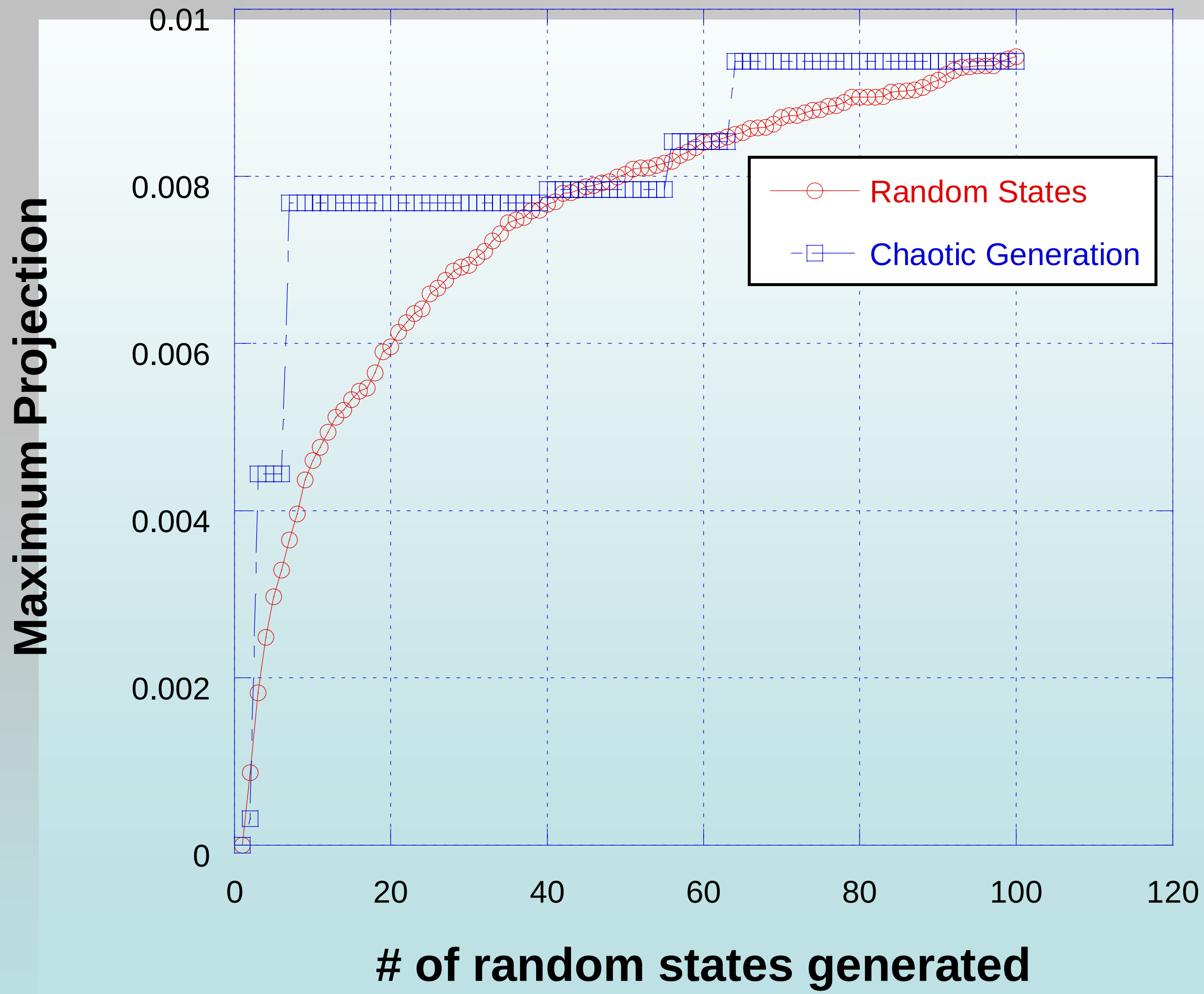
- Random states are used in, remote state preparation, and noise estimation in fault tolerant quantum computing.
- Quantum Chaotic Unitaries typically belong to classes of random matrices.

Generation of Pseudo-random States Using Quantum Chaos



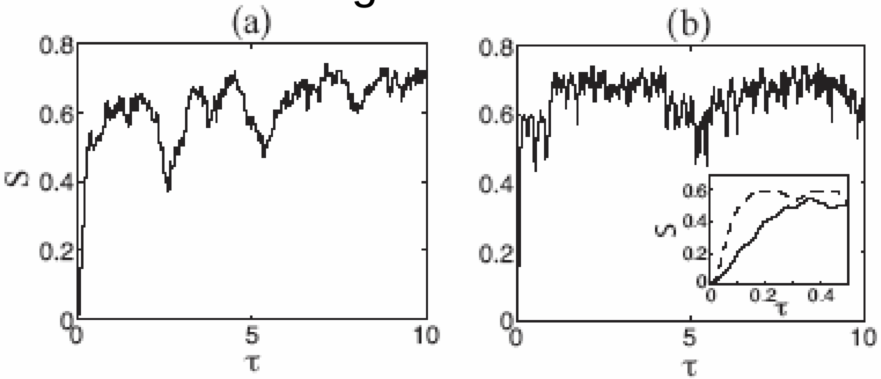
Simply feed a state into a sequence of chaotic time evolution operations, and you end up with a state that approximates a randomly distributed state.

Chaotic Generation of Random States



Entanglement Generation

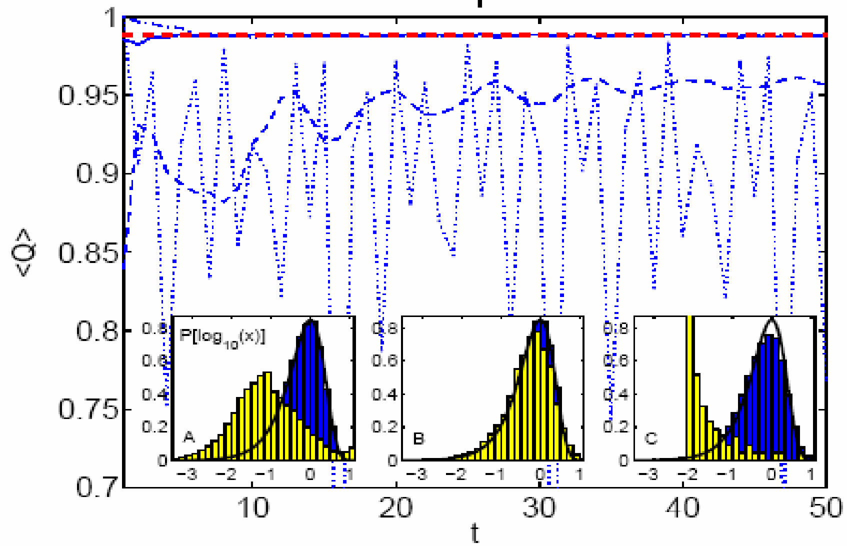
Linear Entropies for Optical Magnetic Lattice



Non-Chaotic Chaotic

Ghose + Sanders (2004)

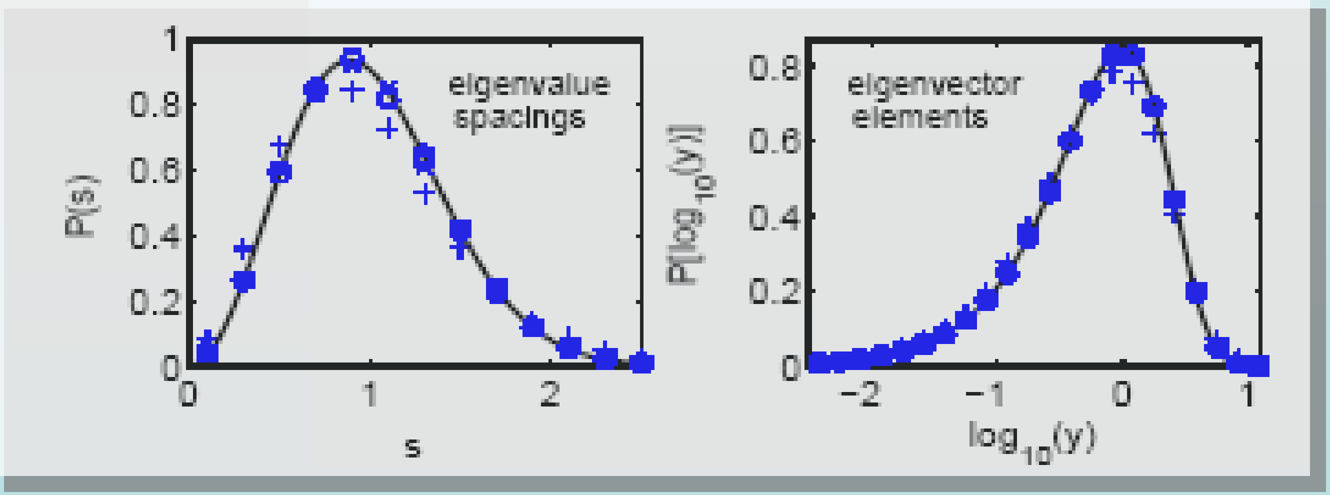
Linear Entropies for Quantum Harper's Map and Saw-tooth Map



Weinstein + Helberg (2005)

Random Matrix Theory

- Chaotic time evolution is complicated, in fact in many cases it is nearly random.
- Time evolution operator similar to that of a random matrix with same symmetry group.



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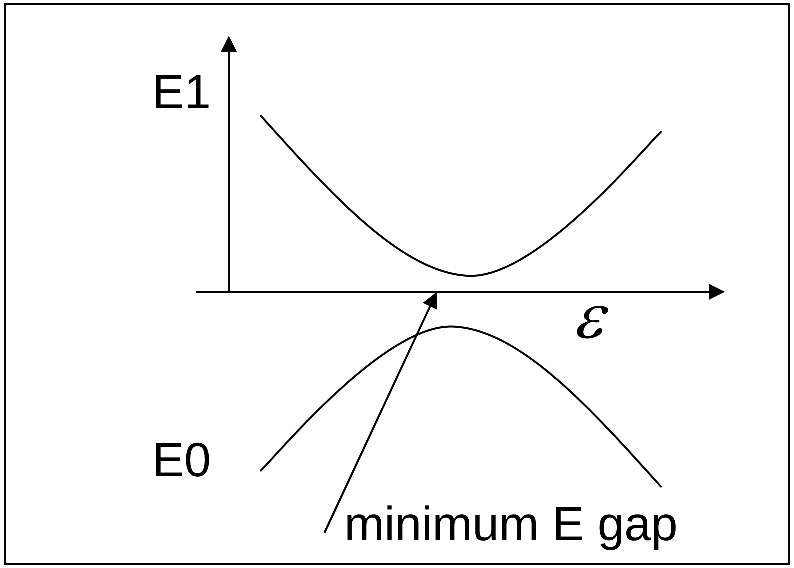
Random Matrix Theory

- Non-Chaotic Hamiltonians do not typically obey random matrix theory.
- Compliance with RMT is from level repulsion

Perturbation theory Predicts that energy levels are repelled under these assumptions

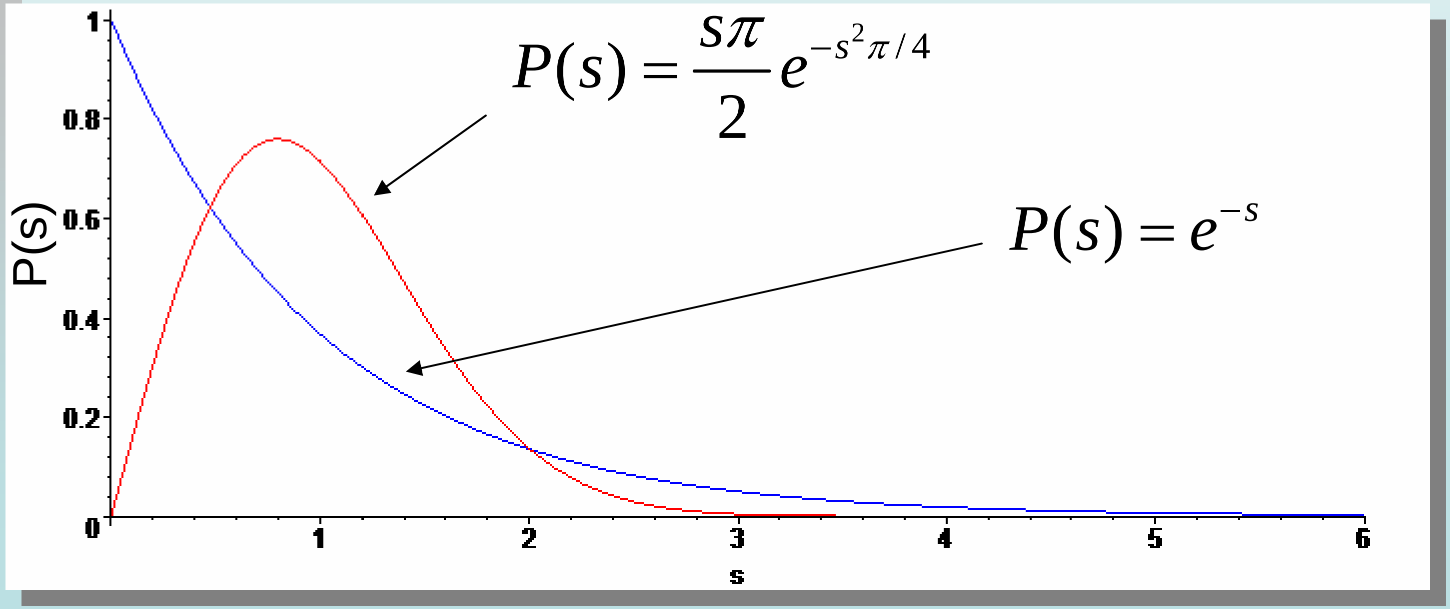


A perturbation applied to make the levels of H_0 degenerate

$$H = H_0 + \varepsilon H_1$$

Random Matrix Theory

- Regular Hamiltonian levels can cross, leading to a poison distribution.
- Level repulsion leads to a Wigner distribution depending on the symettry class of the Hamiltonian.
- The result for time-reversal symmetric systems is below.



Random Matrix Theory and Entanglement

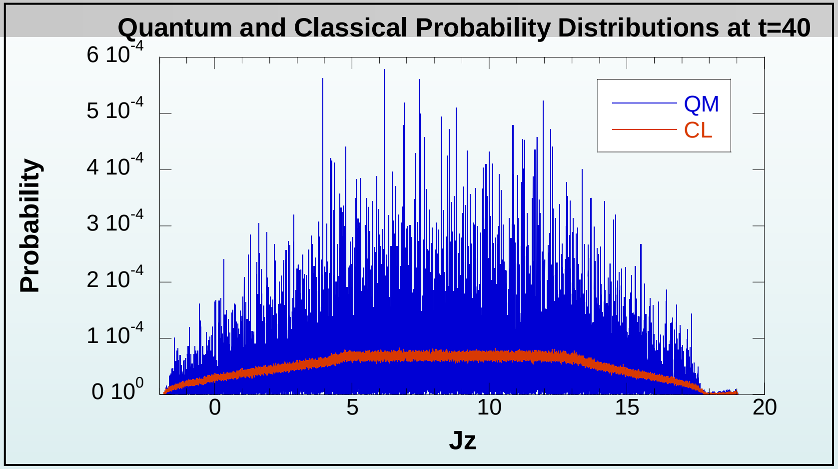
- Random matrix theory predicts the average linear entropy for chaotic systems is :

$$< Q >= \frac{N-2}{N+1}$$

- The entanglement approaches 1 as the dimension of the Hilbert space increases.
- To get the most entanglement, a large Hilbert space must be used.

Downside to chaotic generation

- Chaotic Systems could be more sensitive to small measurement errors.



- Larger Hilbert spaces tend to be more susceptible to decoherence.

How Do Chaotic maps respond to coarse graining in time?

How repeatable is a chaotic map?

Is this entanglement in a useful form?

Measurement

- Can either measure a system at discrete times, or continuously.
- Continuous measurement is often done using a stochastic Schrödinger equation.
- I will focus on discrete measurement, because continuous measurement is analogous to decoherence.

Von Neuman Measurement Model

$$\left|\psi\right\rangle = \sum_{k,m} a_{k,m} \left|k\right\rangle \otimes \left|\alpha_k , m\right\rangle$$

system

Indicator

Other Eigenvalues

Coarse Graining Filter

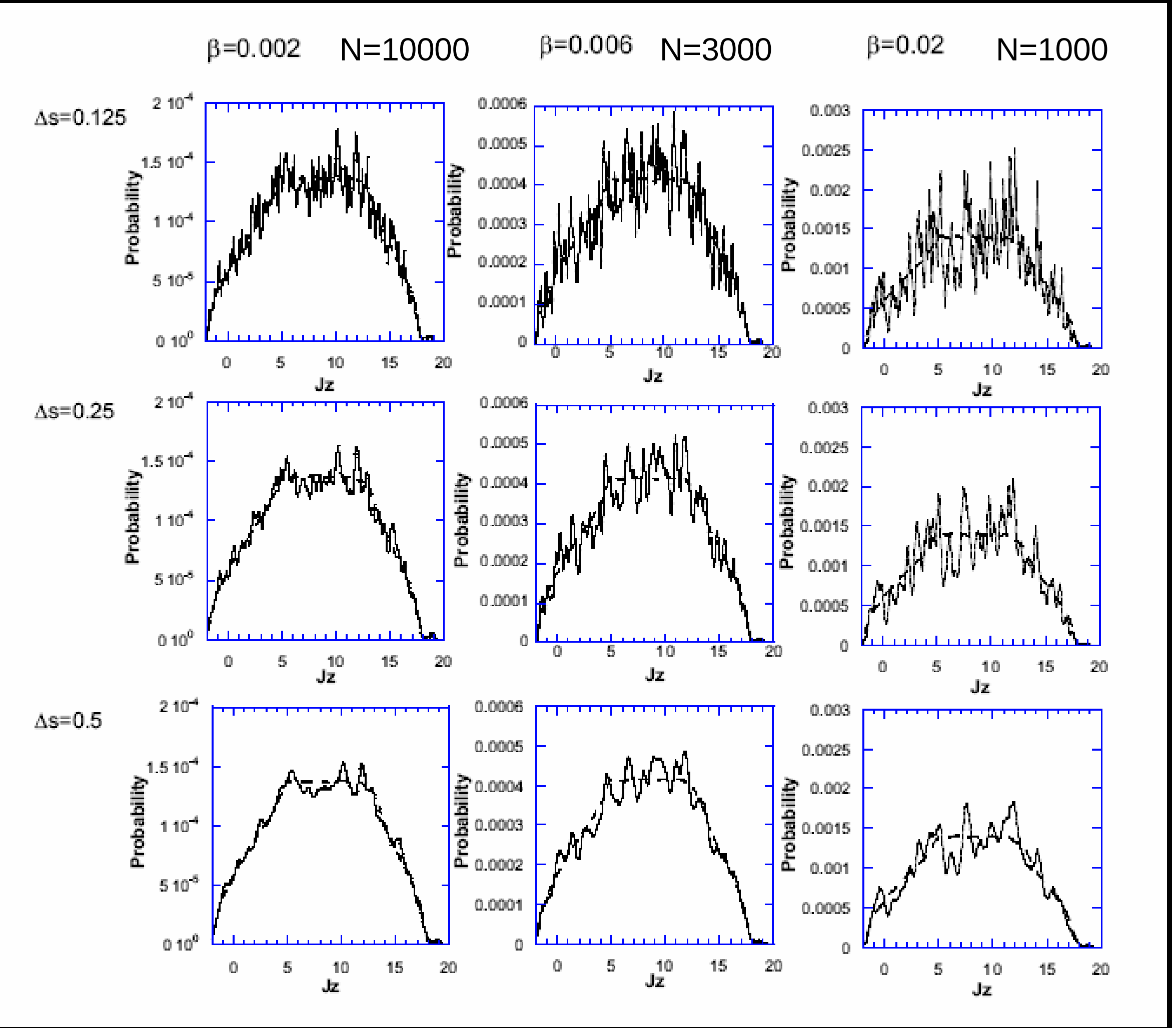
Assumes perfect measurement is obtained out of the system.

$$\rho = \sum_{\Delta=-\infty}^{\infty} \sum_{k,m,k'm'} a_{k,m} a^*_{k',m'} f(\Delta) \left|k\right\rangle \otimes \left|\alpha_{k+\Delta}, m\right\rangle \left\langle k'\right| \otimes \left\langle \alpha_{k'+\Delta}, m'\right|$$

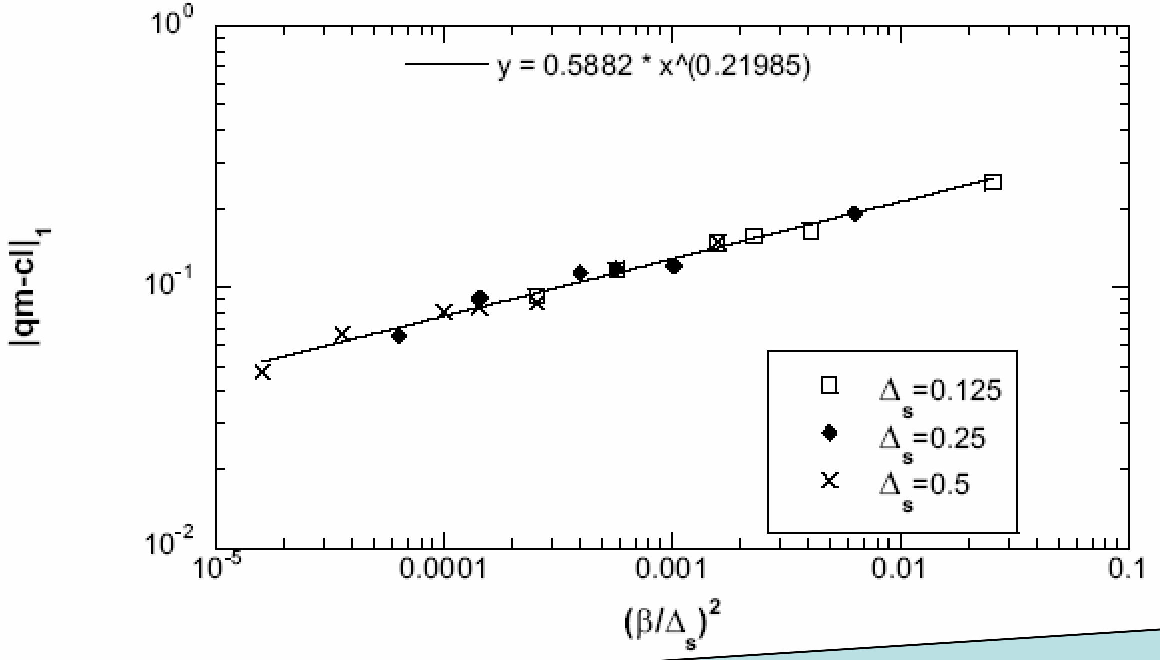
Coarse Graining causes the pure state above to be transformed into a mixed state

Coarse Grained Measurements on Probability Distributions

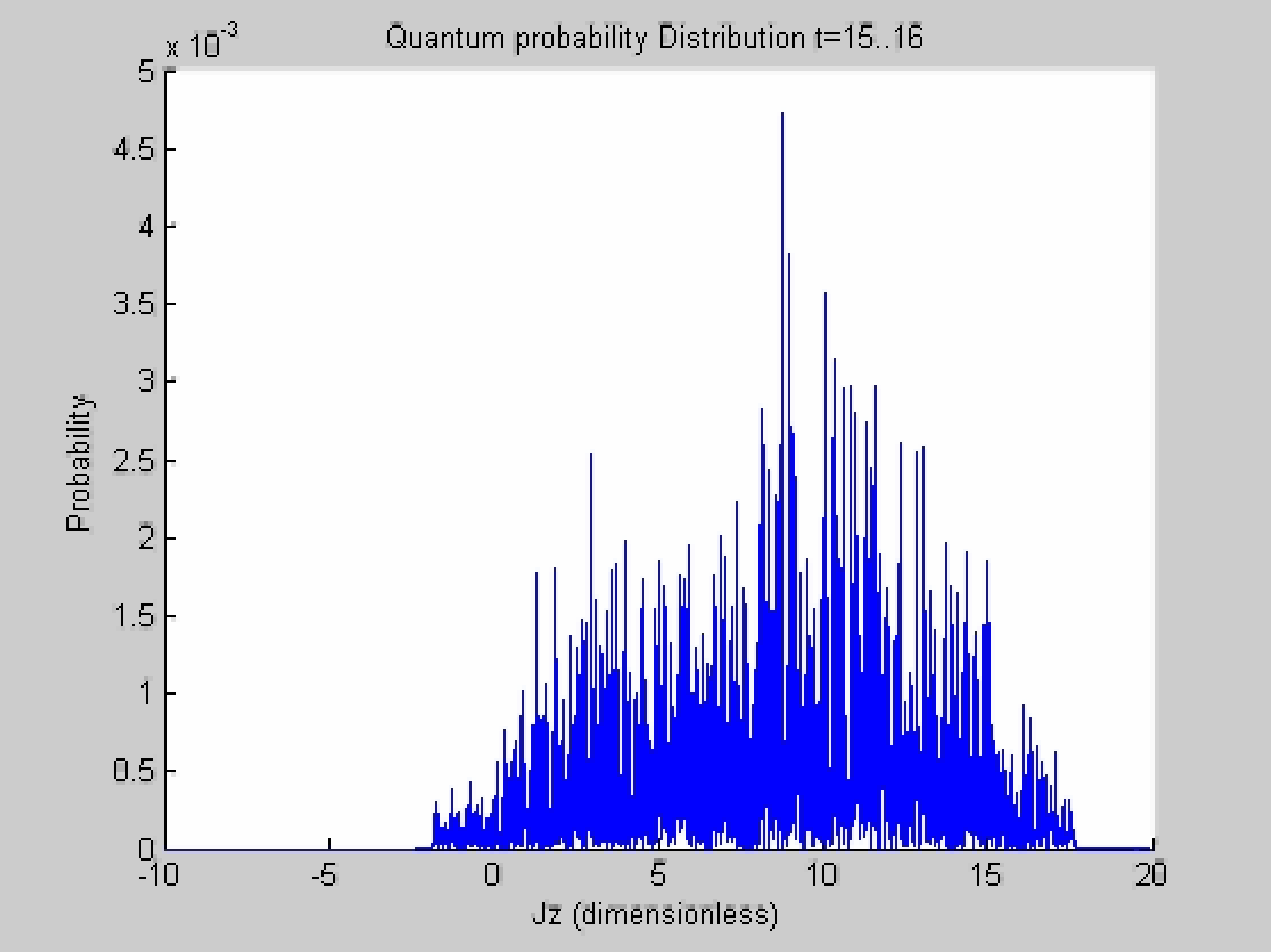
- Distributions become more classical after coarse graining.
- The Larger the Hilbert space the more accurate measurements must be.



How Sensitive are Systems to Coarse Graining?



Quantum Classical differences obey a power-law in resolution and size of Hilbert space.


Quantum probability Distribution t=15..16
x 10^-3
5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
Probability
-10
-5
0
5
10
15
20
Jz (dimensionless)

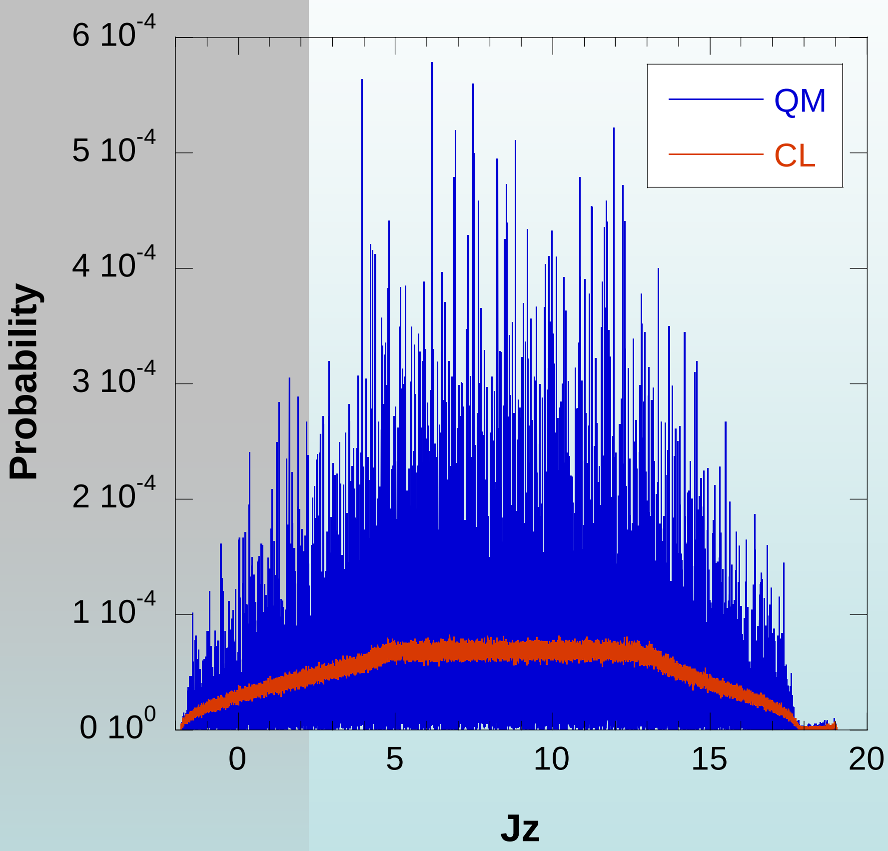
Coarse Graining Temporal Probability Distributions

- Measurements are never perfect, and in reality any measurement occurs over a finite width in time.

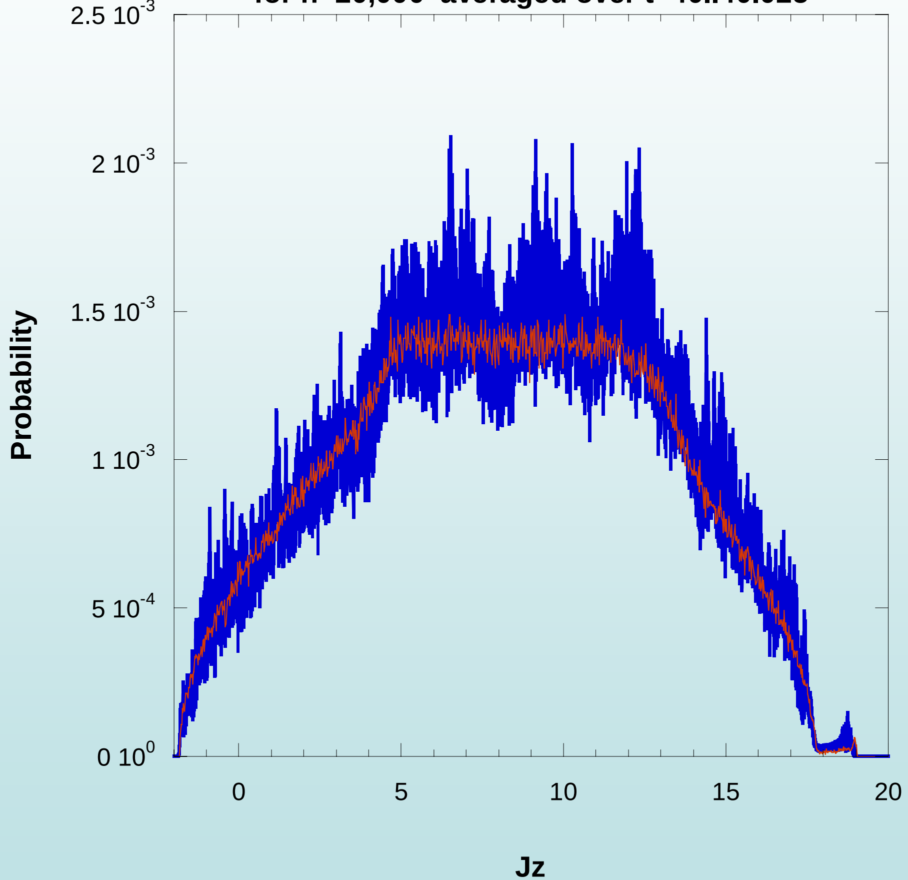
$$P(t;\tau) = \int_{t-\tau/2}^{t+\tau/2} P(\tau)F(t-\tau)d\tau$$

- The observed probability distribution can be found by a convolution of its self with a filter function.

Effects of Coarse Graining in time



Quantum and Classical Probability distribution at t=40,
for n=20,000 averaged over t=40..40.025



- Probability distributions for a quantum chaotic system look more classical after coarse graining.

How to Model Coarse Graining in a Quantum Setting

- Measurements made at different times are independent of each other.
- Each measurement is projective

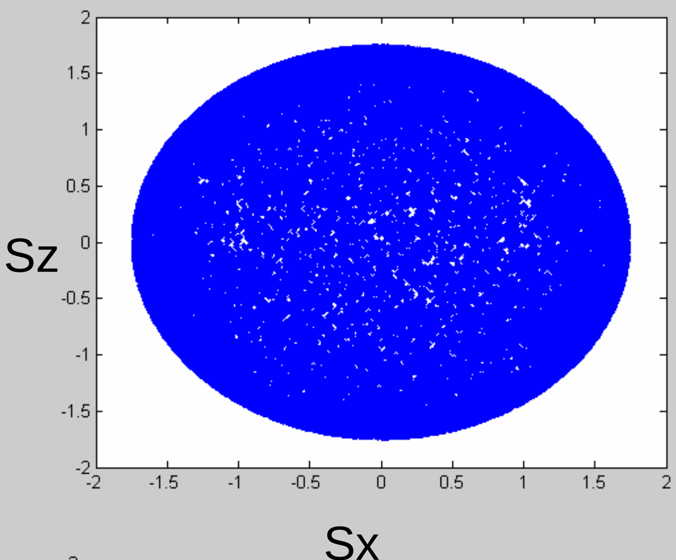
$$\rho(t;\tau) = \int_{t-\tau/2}^{t+\tau/2} \rho(\tau) F(t;\tau) d\tau$$

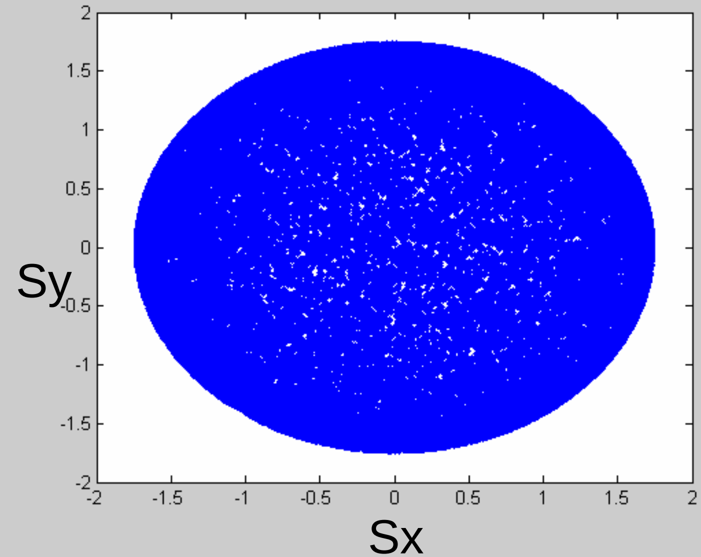
- Here F is a filter function of unit weight.

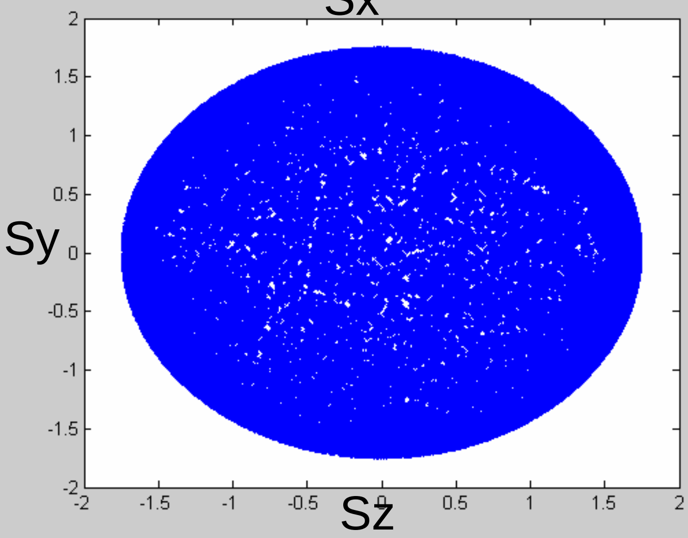
Poincare Sections

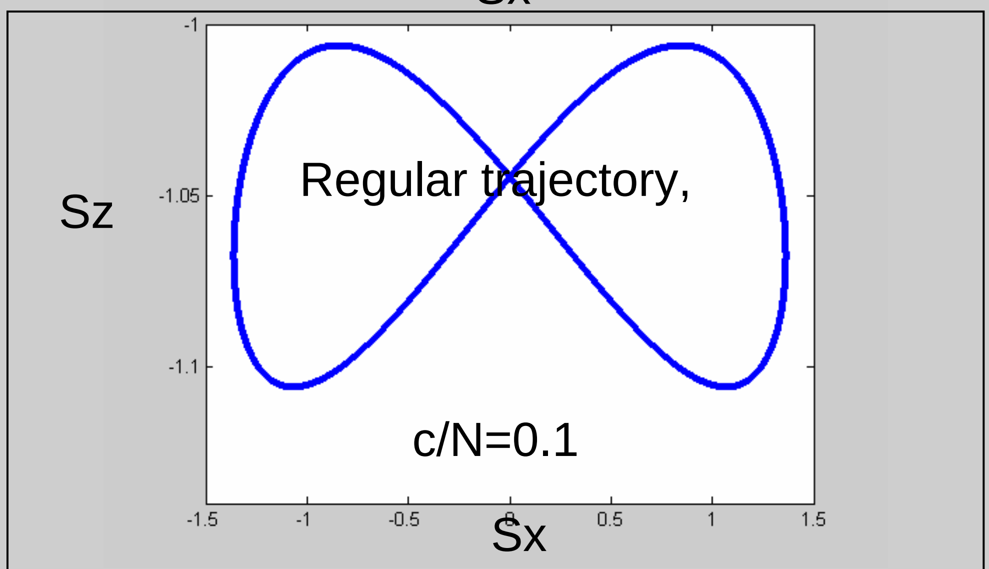
Stroboscopic views of 10,000 kicks for the classical map for the kicked spins.

Lack of regular torii suggests that it is chaotic.



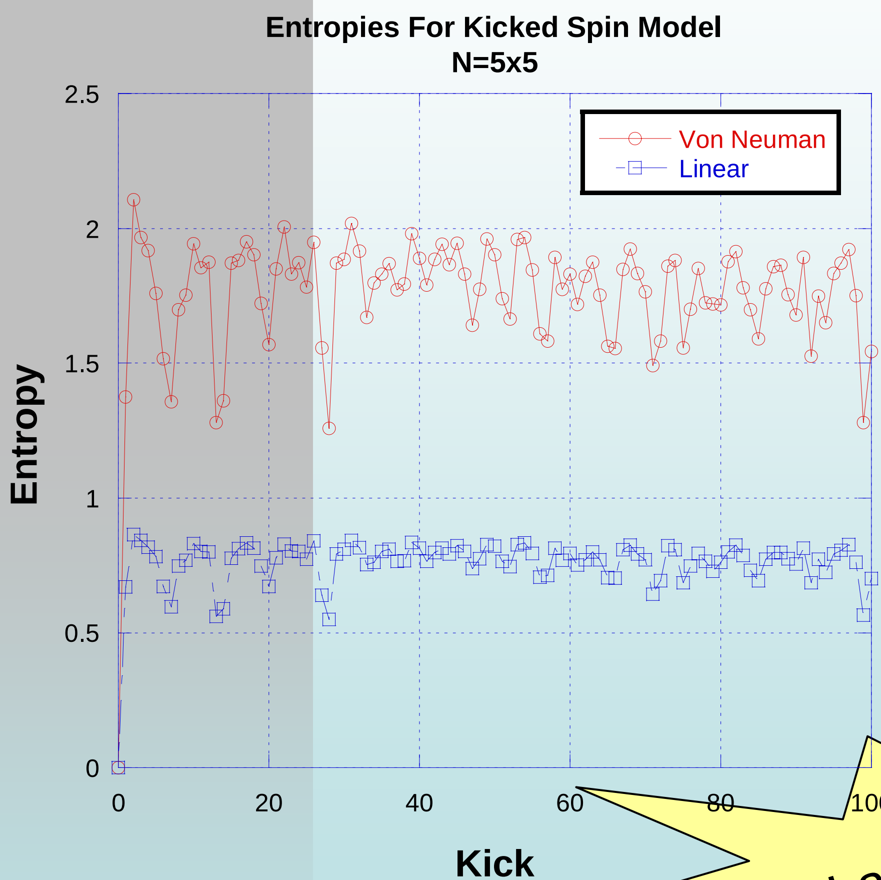


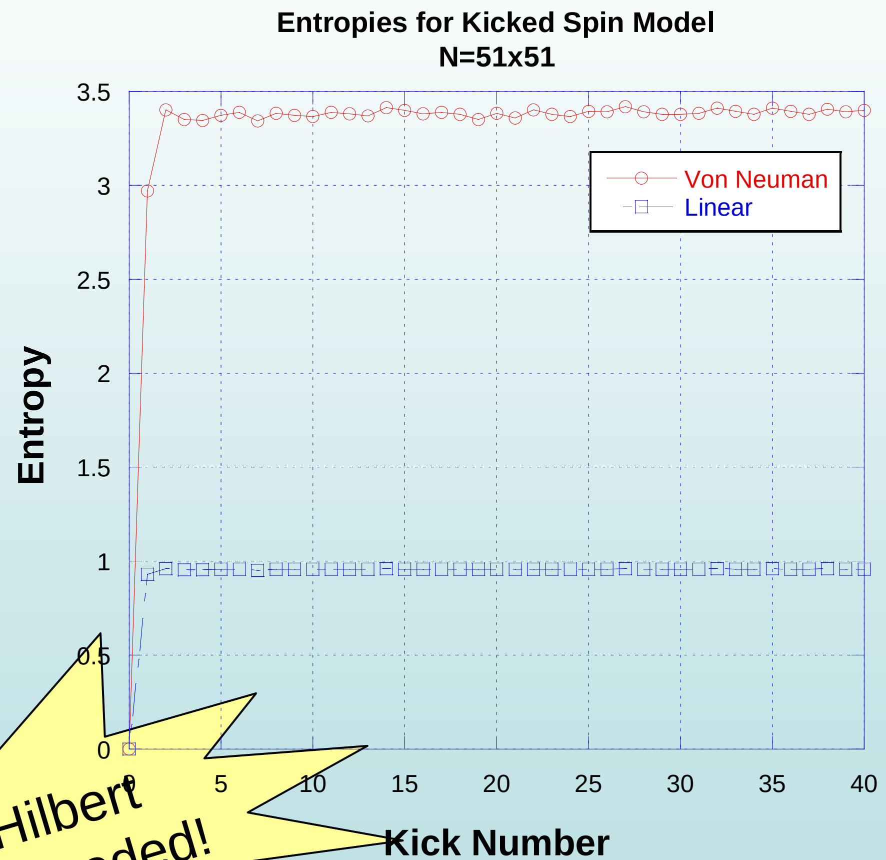




Results For Kicked Spin

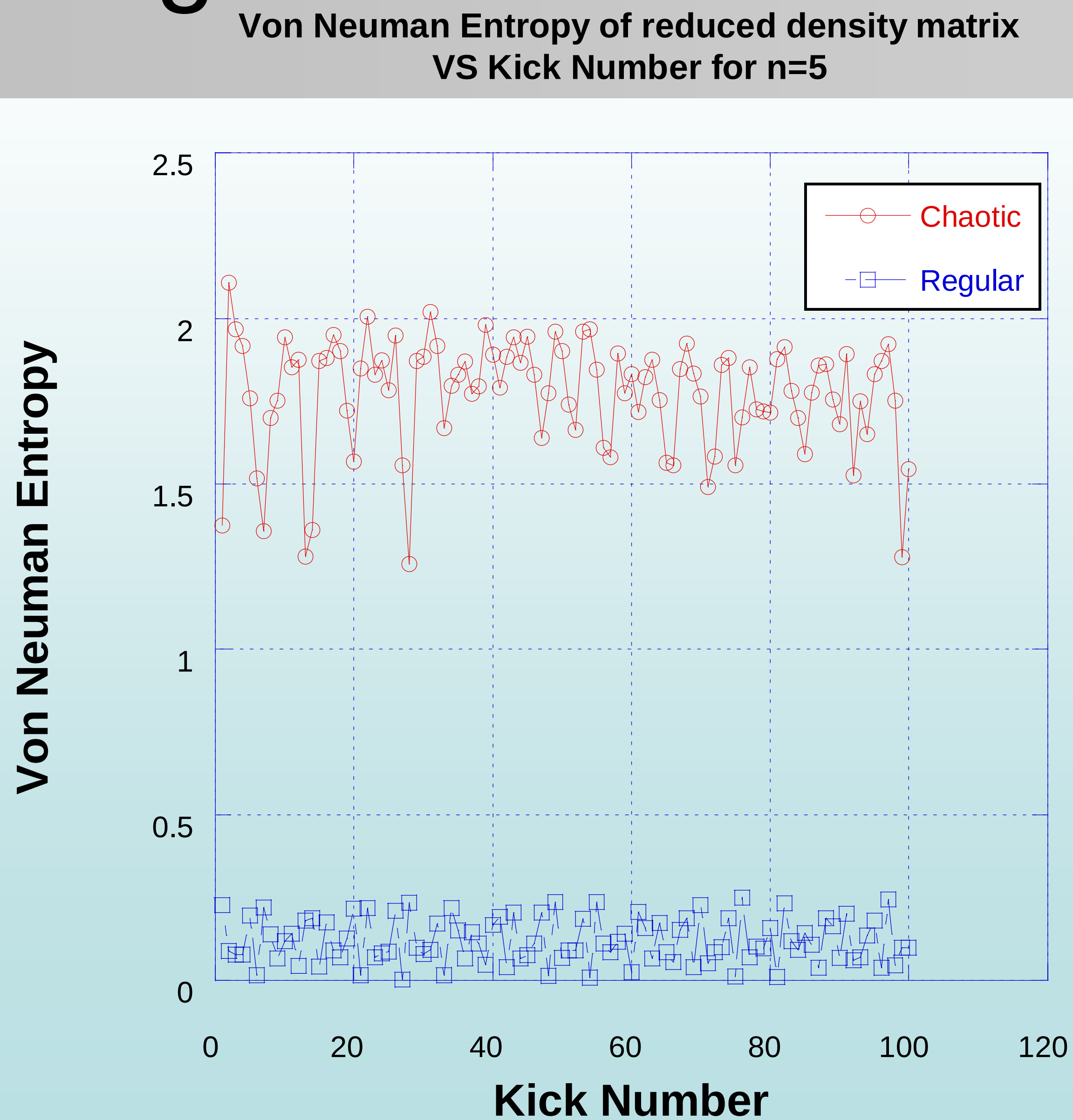
a=5, c/N=2.8



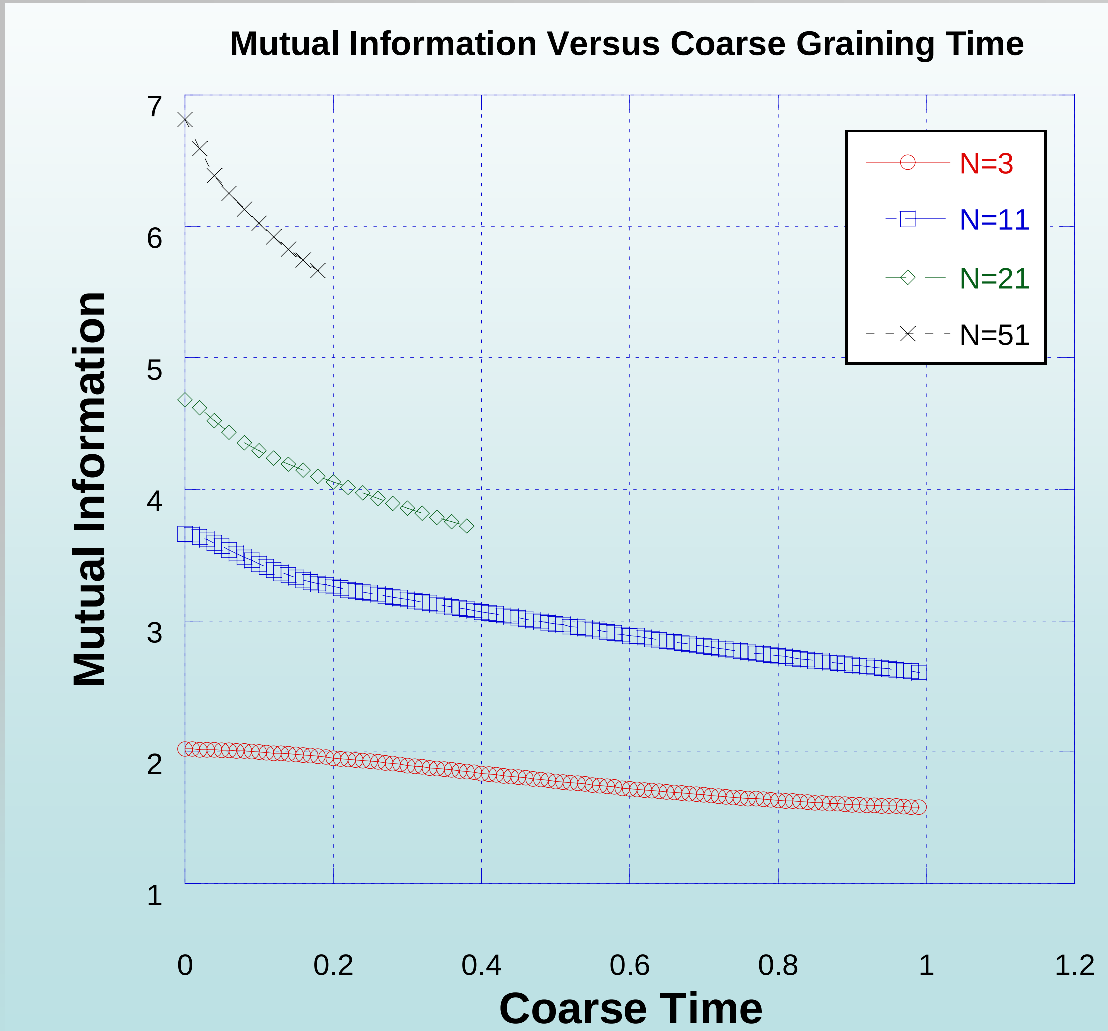


A Large Hilbert Space is needed!

Regular Versus Chaotic



What Does Coarse graining do to entanglement?



- Mutual information rapidly decays for higher spins.

Observation Errors

- The sensitivity of quantum chaos to perturbations can limit its usefulness.
- The perturbation sensitivity applies also to non-chaotic sensitivity…

Purely Quantum: 1)Hilbert space too small to generate much entanglement. 2)Entanglement varies rapidly from period to period	Hilbert space is ‘JUST RIGHT’ for quantum chaos to generate entanglement.	Semi classical: 1)Lots of entanglement generated 2) decoherence and measurement errors render it useless.

Further Questions About Quantum Chaos

- Is the Von Neuman Entropy/Linear entropy measure the usable entanglement for chaotic systems?
- How do singlet fractions vary when measurement errors occur?
- Can quantum chaotic evolutions be used in cryptography?

Conclusions

- Quantum Chaotic operators can approximate random states.
- They also produce entanglement far faster than regular systems.
- Sensitivity to measurements limits the utility of quantum chaos.