Mode Theory of Photonic Qubits in Quantum Information

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Outline

- Introduction
- Paraxial Approximation
- Mode Overlap
- Hong-Ou-Mandel Dip
- Quantum Fingerprinting
- Conclusions

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Introduction

Maxwell's Equations

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Introduction

$$\nabla^2 \vec{A} - \partial_t^2 \vec{A} = 0$$

$$\vec{A}(\vec{x},t) = \int d^3k \sum_{\sigma=1,2} A_{k\sigma} \vec{\varepsilon}_{k\sigma} N_{k\sigma} \exp(-i\omega_k t)\exp\left(i\vec{k}\cdot\vec{x}\right) + c.c$$

$$\vec{A}^{(+)}(\vec{x},t) = \int d^3k \sum_{\sigma=1,2} \hat{a}_{k\sigma} \vec{\varepsilon}_{k\sigma} N_{k\sigma} \exp(-i\omega_k t)\exp\left(i\vec{k}\cdot\vec{x}\right)$$

$$A^{(+)}(\vec{x},t) = \sum_n A_n(\vec{x},t)\hat{a}_n$$

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Paraxial Approximation

Consider a light field of a running wave with a spatially and temporally varying envelope.

$$\vec{E} = E(\vec{x}, t)e^{ikz - i\omega t}\vec{\varepsilon}$$

Assume the envelope varies slowly with z compared to e^{ikz} and with t compared to $e^{i\omega t}$.

$$k\partial_z E >> \partial_z^2 E$$

$$\omega\partial_t E >> \partial_t^2 E$$

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Paraxial Approximation

Maxwell's Wave Equation in a vacuum:

$$\left(\frac{1}{c^2}\partial_t^2 - \nabla^2\right)\vec{E} = 0$$

becomes:

$$2ik\left(\frac{1}{c}\partial_t + \partial_z\right)E = -\left(\partial_x^2 + \partial_y^2\right)E$$

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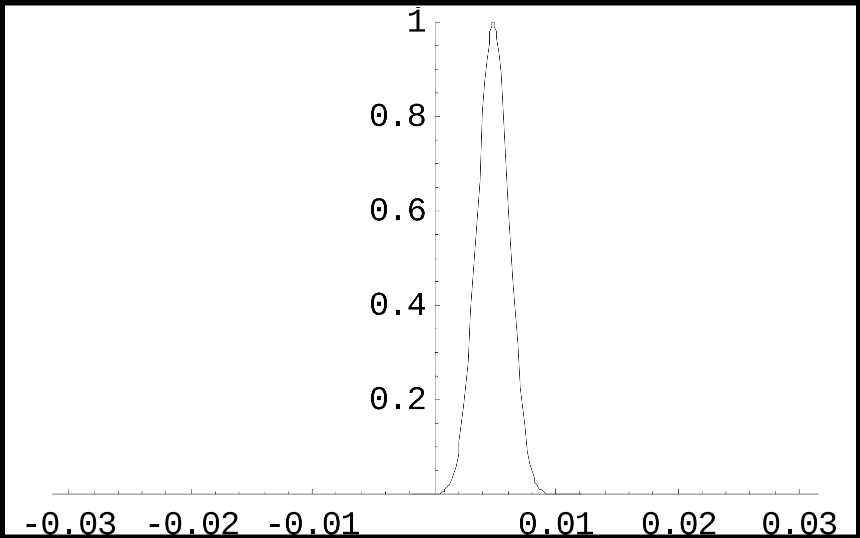
Paraxial Approximation

The equation becomes analogous to the Schrödinger equation and can be solved with a Gaussian

A light beam approaching a medium would look like the following:



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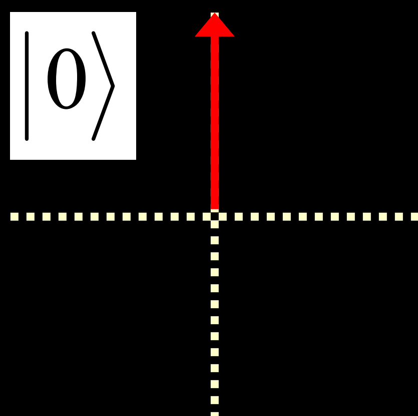
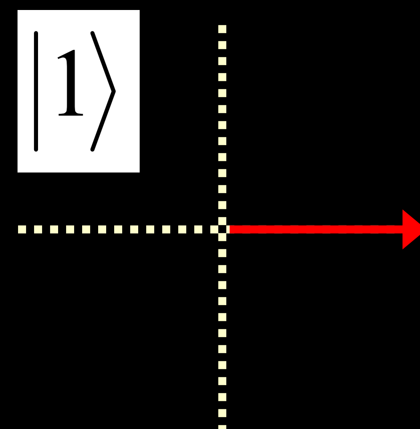
Mode Overlap

- Consider Photons:





Photonic qubits offer a convenient way to represent information, utilizing their polarization degree of freedom.

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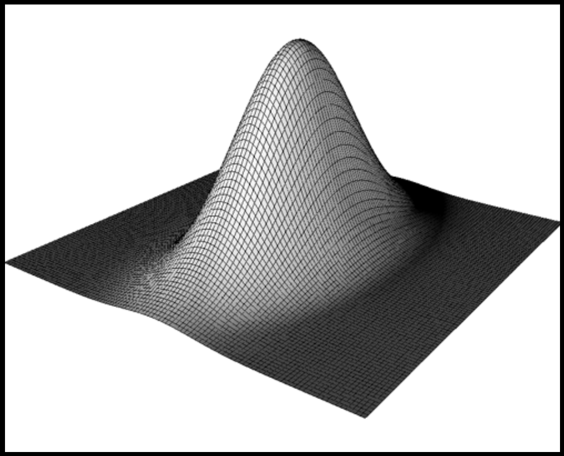
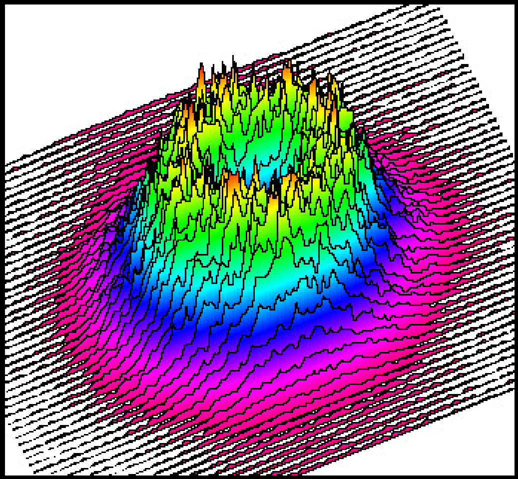
Mode Overlap

However, it is not this simple. A photonic qubit has a finite spatial extent that must be considered.



Gaussian Mode



Laguerre-Gaussian Mode

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Mode Overlap

Harmonic oscillator algebra is very useful for describing photons and photon modes. Basically, creation and annihilation operators ‘create’ and ‘destroy’ photons from a particular state.

$$\hat{a}(\hat{a}^{\dagger}|0\rangle) = |0\rangle$$

$$\hat{a}^{\dagger}|0\rangle \longrightarrow$$

$$\hat{a}$$

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Mode Overlap

If a photon mode is orthogonal to another photon mode, they will obey the following relation.

$$[\hat{a}_m, \hat{a}_n^\dagger] = \delta_{m,n}$$

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Mode Overlap

$$A^{(+)}(\vec{x},t) = \sum_n A_n(\vec{x},t)\hat{a}_n$$

$$\left[\hat{a}_n, \hat{a}_m^\dagger\right] = \left\langle \vec{A}_n \middle| \vec{A}_m \right\rangle$$

However, if there is spatial overlap in the photon modes, the commutator of the creation and annihilation operator is represented by the inner product of the two photon modes. They are *not* orthogonal modes.

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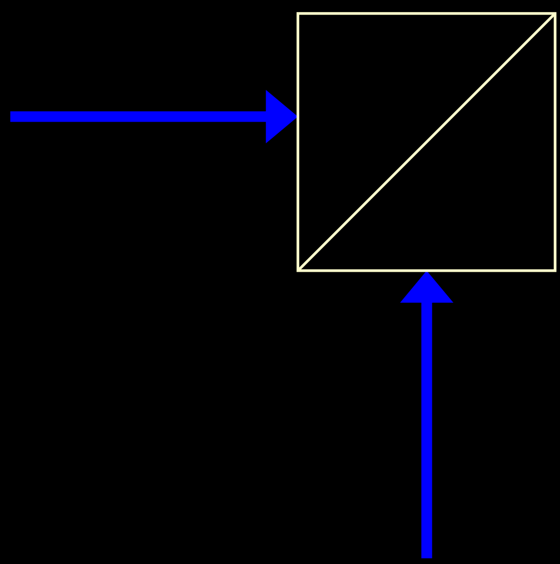
Hong-Ou-Mandel Dip



$$\rho_{in} = \rho_{A,in} \otimes \rho_{B,in}$$

$$\rho_{A,in} = \int d\tau f_A(\tau) \left|1_{A,\phi(\tau)}\right\rangle\left\langle 1_{A,\phi(\tau)}\right|$$

$$\rho_{B,in} = \int d\tau' f_B(\tau') \left|1_{B,\psi(\tau')}\right\rangle\left\langle 1_{B,\psi(\tau')}\right|$$

$$U_{BS}\left|1_{A,\phi}, 1_{B,\psi}\right\rangle = \frac{1}{2}\left(\hat{a}^{\dagger}_{A,\phi}\hat{a}^{\dagger}_{B,\psi} - \hat{a}^{\dagger}_{A,\psi}\hat{a}^{\dagger}_{B,\phi}\right)\left|0\right\rangle + \frac{1}{2}\left(\hat{a}^{\dagger}_{B,\phi}\hat{a}^{\dagger}_{B,\psi} - \hat{a}^{\dagger}_{A,\phi}\hat{a}^{\dagger}_{A,\psi}\right)\left|0\right\rangle$$

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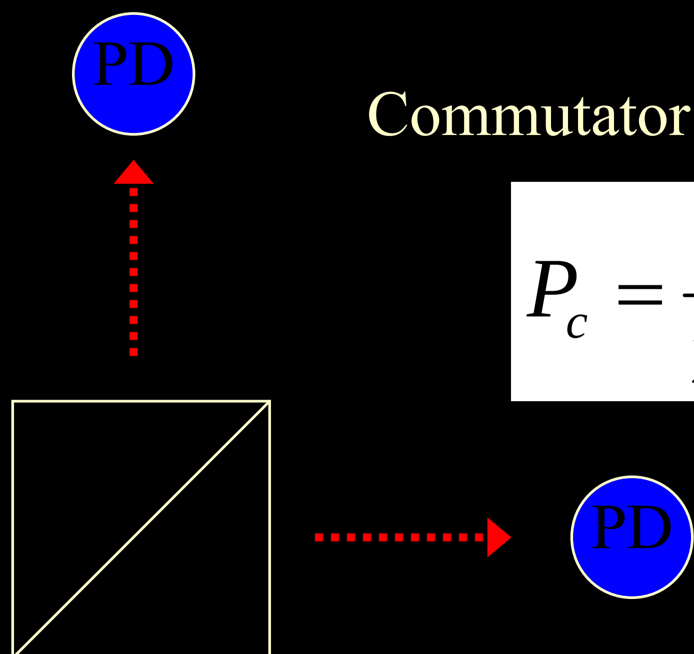
Hong-Ou-Mandel Dip

Coincidence Probability:

$$P_c = \frac{1}{4}\left(2 - \left[\hat{a}_{A,\phi}, \hat{a}^{\dagger}_{A,\psi}\right]\left[\hat{a}_{B,\psi}, \hat{a}^{\dagger}_{B,\phi}\right] - \left[\hat{a}_{A,\psi}, \hat{a}^{\dagger}_{A,\phi}\right]\left[\hat{a}_{B,\phi}, \hat{a}^{\dagger}_{B,\psi}\right]\right)$$



Commutator is defined by mode overlap

$$P_c = \frac{1}{2}\left(1 - \left|\langle \phi(\tau) | \psi(\tau') \rangle\right|^2\right)$$

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Hong-Ou-Mandel Dip

Example using realistic pulses of Gaussian wavepackets

$$\phi(\tau) = \sqrt{\frac{\hbar}{2ck\varepsilon_0 cT\sqrt{\pi}}}\, e^{-\frac{(x-c\tau)^2}{2c^2T^2}} e^{ik(x-c\tau)}\vec{\varepsilon}_\phi$$

$$\psi(\tau) = \sqrt{\frac{\hbar}{2ck\varepsilon_0 cT\sqrt{\pi}}}\, e^{-\frac{(x-c\tau)^2}{2c^2T^2}} e^{ik(x-c\tau)}\vec{\varepsilon}_\psi$$

$$T \approx 170\, fs$$
$$\omega \approx 2.38\times 10^{15}\, s^{-1}$$

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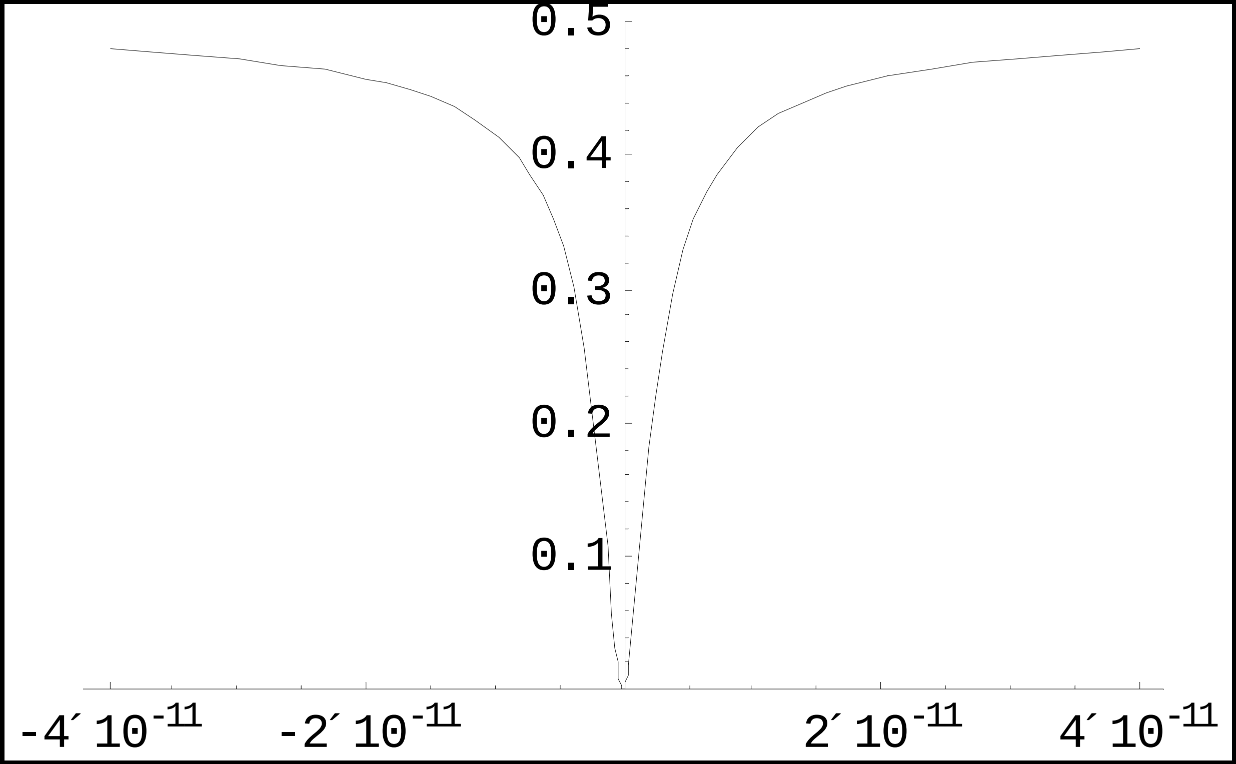
Hong-Ou-Mandel Dip

$$P_c = \frac{1}{2} - \frac{1}{2}\left(\frac{\Delta\tau^2 + 4T^2\left(T^2 + \Delta\tau^2\right)\omega^2}{4T\left(T^2 + \Delta\tau^2\right)^{\frac{3}{2}}\omega^2}\right)\left|\varepsilon_\phi^* \cdot \varepsilon_\psi\right|^2$$



P_c

Width of Mode Mismatch Probability (s)

Hong-Ou-Mandel Dip

What does this mean?

If the mode mismatch distribution is narrow, and the modes are nearly perfectly overlapping the probability of coincidence is nearly zero for identical polarization states.

However, if the mode mismatch distribution is large, the coincidence probability tends to ½ for identical polarization states.

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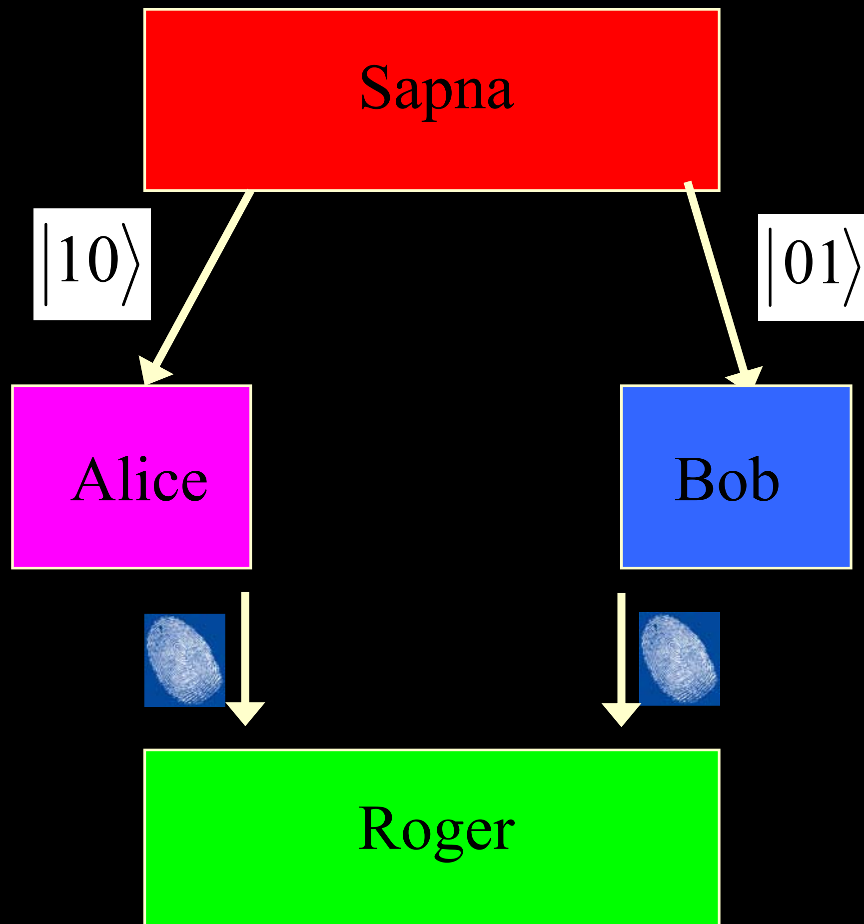
Quantum Fingerprinting



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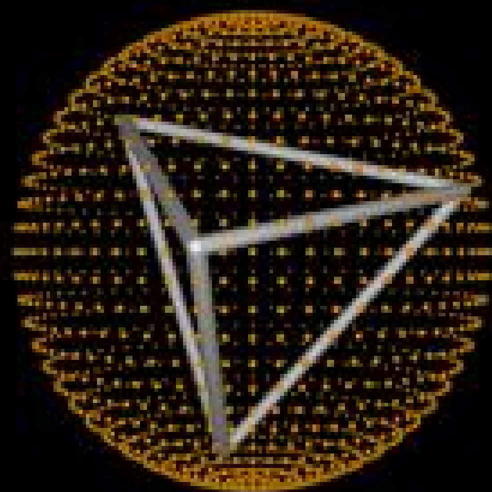
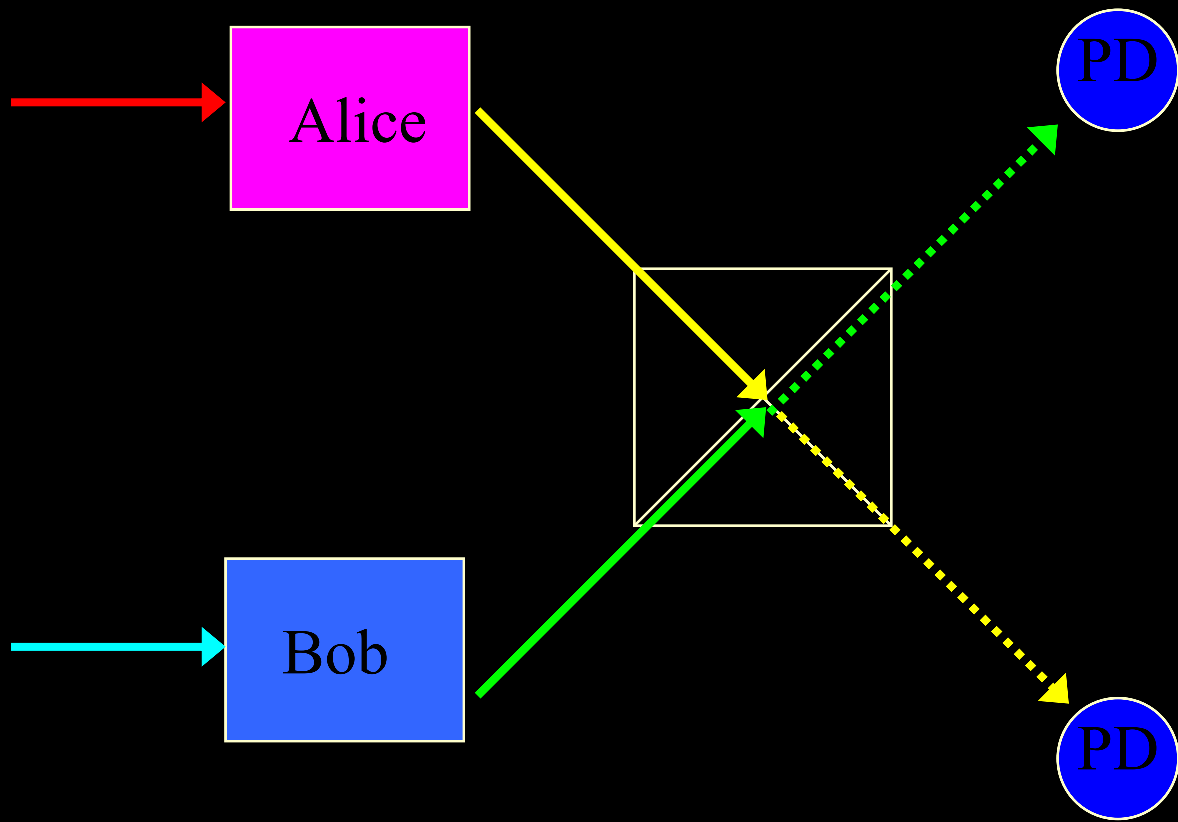
Quantum Fingerprinting

Alice and Bob each map their message photon to a tetrahedral state on the Bloch Sphere. Their photons are assumed to be prepared in the same way.



Quantum Fingerprinting

Linear Optical Setup



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Quantum Fingerprinting

Explanation with Bell States

The natural basis of two photon polarization is:

$$|HH\rangle, |VV\rangle, |HV\rangle, |VH\rangle$$

The Bell States can thus be used as a basis:

$$\frac{1}{\sqrt{2}}\left(|HH\rangle + |VV\rangle\right), \frac{1}{\sqrt{2}}\left(|HH\rangle - |VV\rangle\right), \frac{1}{\sqrt{2}}\left(|HV\rangle + |VH\rangle\right), \frac{1}{\sqrt{2}}\left(|HV\rangle - |VH\rangle\right)$$

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Quantum Fingerprinting

Assume each photon is in the same state:

$$\left(\alpha\left|0\right\rangle+\beta\left|1\right\rangle\right)\otimes\left(\alpha\left|0\right\rangle+\beta\left|1\right\rangle\right)$$

This would yield the following:

$$\alpha^2\left|00\right\rangle+\beta^2\left|11\right\rangle+\alpha\beta\left(\left|01\right\rangle+\left|10\right\rangle\right)$$

This linear combination of Bell States does not contain the state:

$$\left|\psi^-\right\rangle\frac{1}{\sqrt{2}}\left(\left|10\right\rangle-\left|01\right\rangle\right)$$

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Quantum Fingerprinting

Why is this significant?

The $|\psi^-\rangle$ Bell State is unchanged by the beam splitter, and also will cause a coincidence detection. Thus, a coincidence detection implies the photon pair had a component in the $|\psi^-\rangle$ direction. The other three Bell-States will exit through the same port of the beam splitter

So, if there is a coincidence detection, the photons were unequal since it would contain a $|\psi^-\rangle$ component that is not present if the photons were equal.

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Quantum Fingerprinting

When there is zero delay in the interferometer, there is spatial overlap of the photon modes, thus creating a HOM dip.

When the incident photons are in the same spectral state, the ... coincidence rate. This corresponds to the messages possibly being equal(The photon modes overlap, crea

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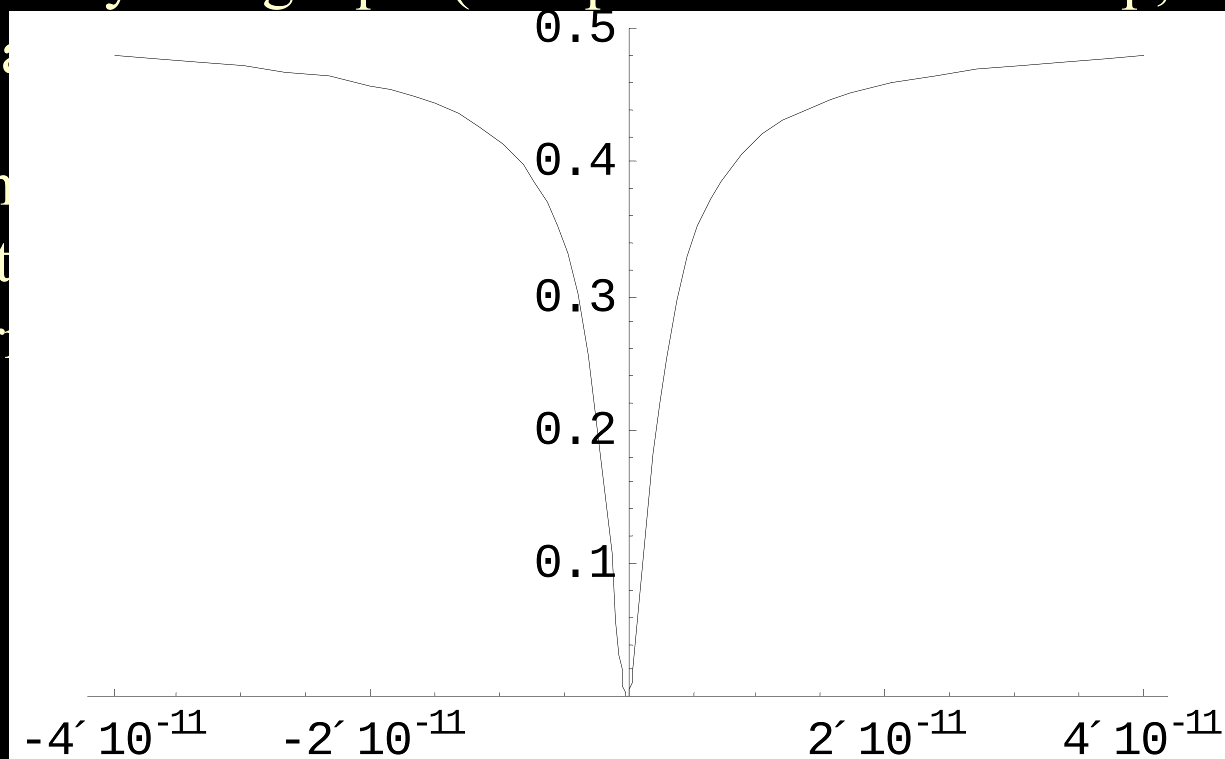

Conclusions

Solutions to the equations of light give the photons a finite spatial extent. This spatial extent may overlap creating a non-orthogonality associated with polarization encoded 0's and 1's.

This attribute of photons can be harnessed or avoided depending on what is required.

There are more things in 0's and 1's, Horatio, than are dreamt of in your philosophy.--Hamlet