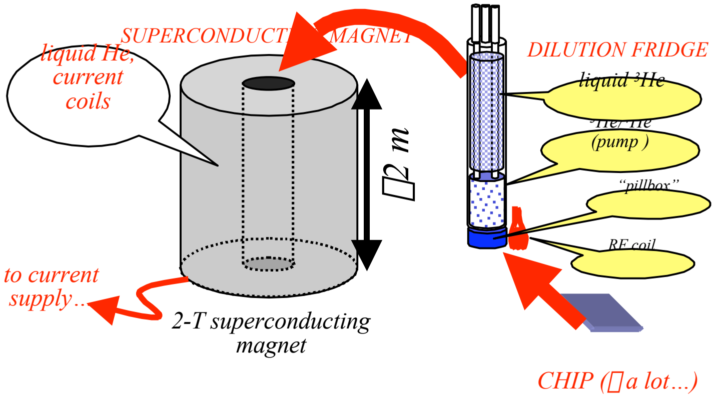
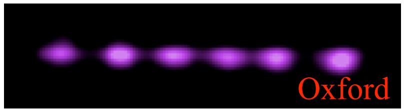
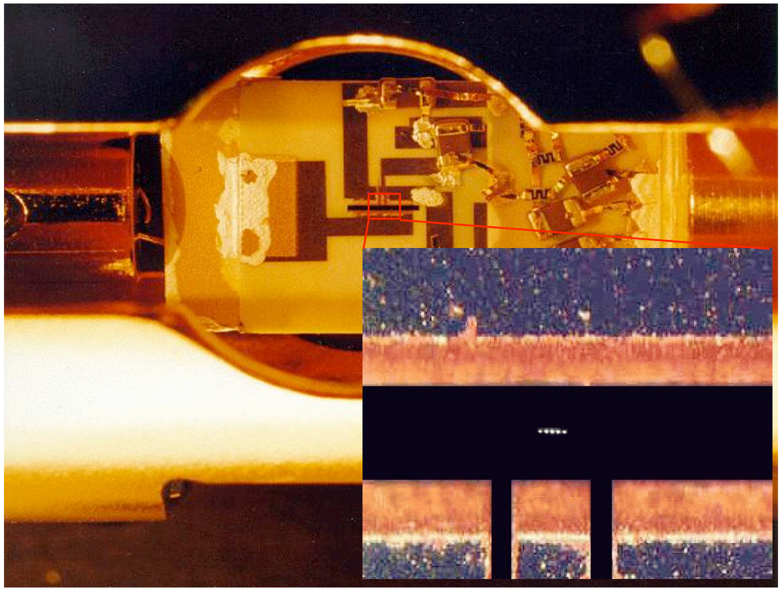
Physical "Implementations" of Quantum Computing







PIMS/MITACS summer school– June, 2003

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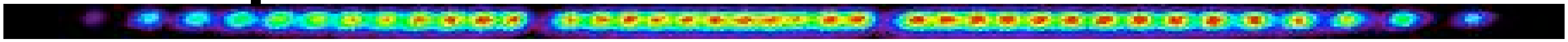
http://physserv.mcmaster.ca/~kingb/King_B_h.html

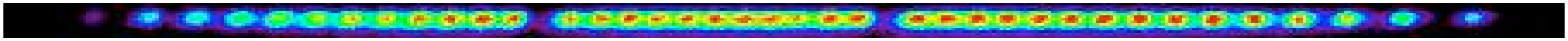
Frontspiece:



"Quantum mechanics
does not occur in a
Hilbert space – it occurs
in a laboratory!"

- Asher Peres

Outline:



- propaganda ✓

Part I:

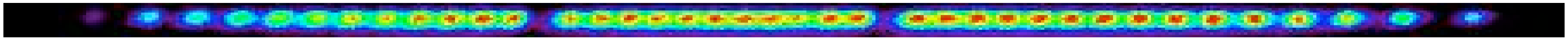
  - quantum behaviour
  - quantum dynamics
  - “real” qubits (by way of atoms…)
  - physical requirements
  - “rogue’s gallery” of architecture proposals

Part II:

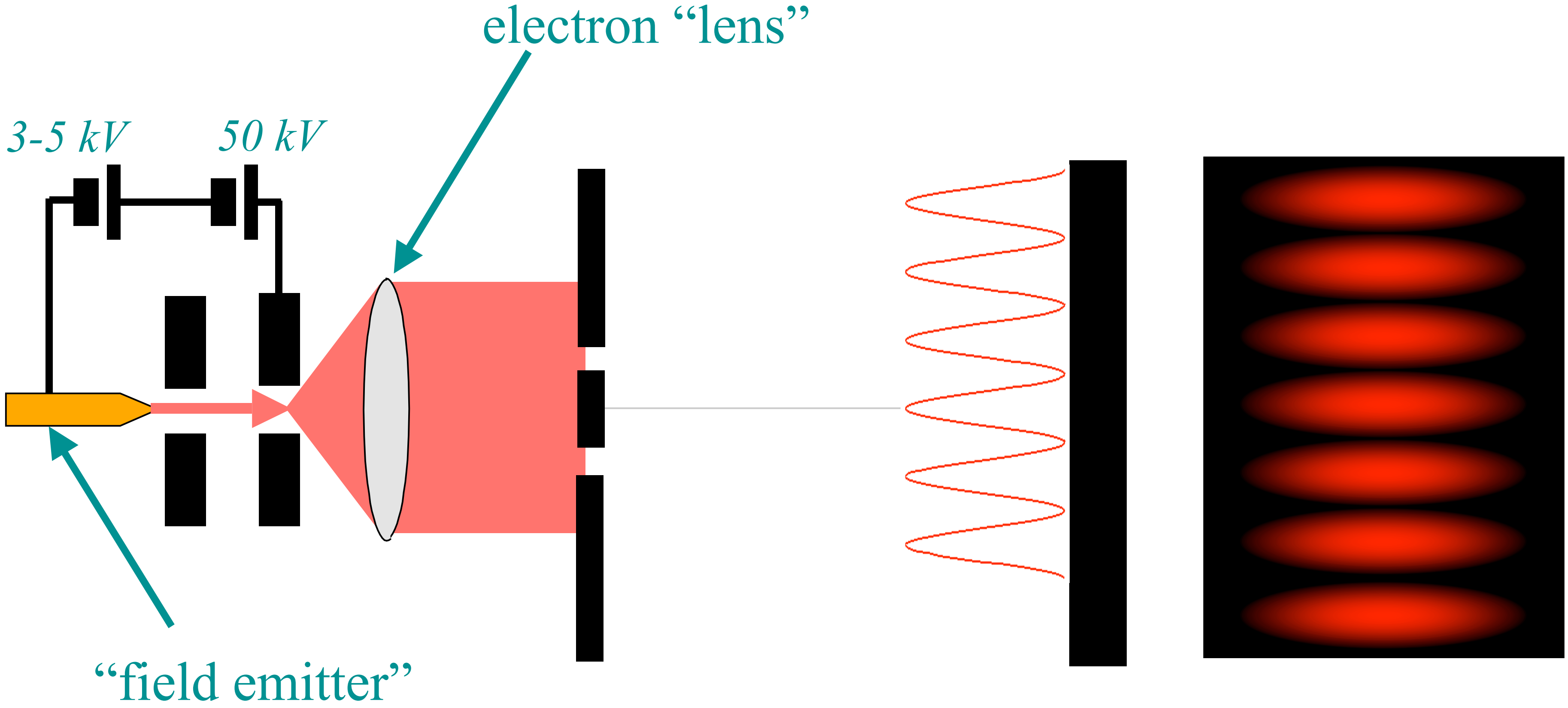
  - ion traps
  - atomic qubits, again
  - logic gates

- the future (?…)

Electrons and holes!:



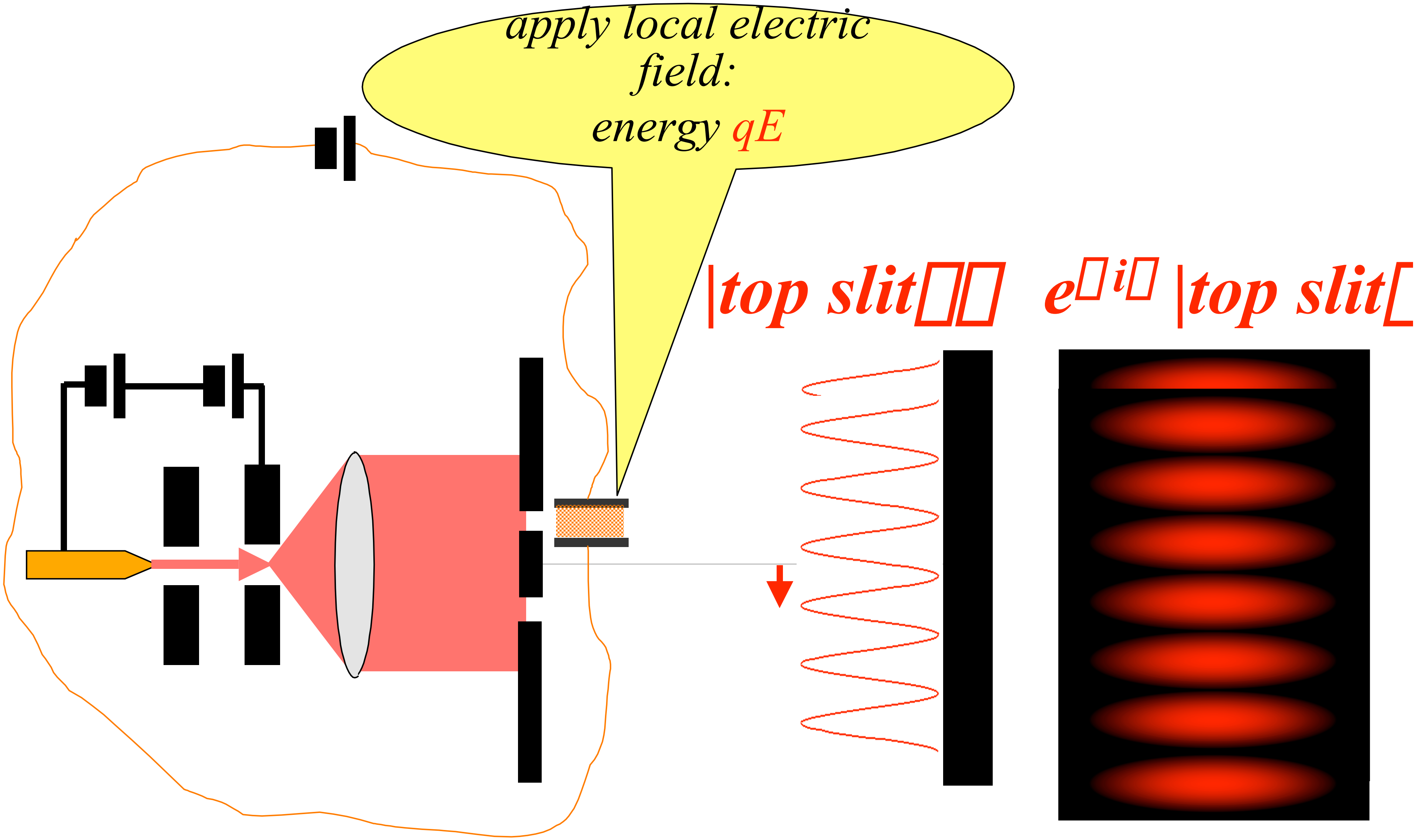
A Tonomura, *et al.*, 1989 Am. J. Phys. **57**, 117-120 (89); L Marton, *et al.*, Phys. Rev. **90**, 490-491 (53); C Jönsson Zeit. Phys. **161**, 454-474 (61)



$$\textit{state of electron} \sim |\textit{top slit}\rangle + |\textit{bottom slit}\rangle$$

Electrons and holes!:

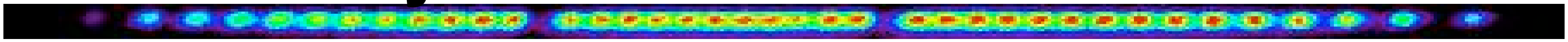
- locally change part of the electron's energy…



apply local electric field:
energy qE

$|\textit{top slit}\rangle \rightarrow e^{-i\phi}|\textit{top slit}$

Quantum dynamics:



- the Schrödinger equation:
  - tells us how the quantum state *changes in time*

$$i\hbar \frac{d}{dt}|\psi\rangle = H|\psi\rangle$$

quantum state (state vector)

Hamiltonian operator

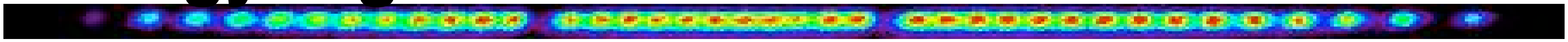
•*energy* of quantum state

- energy *eigenstates*:

$H|\psi_{E,n}\rangle = E|\psi_{E,n}\rangle$ *E is the energy of* $|\psi\rangle$

  - *state vector is unchanged (up to a constant…)*

Energy Eigenstates:



- *make solving the Schrödinger eq. easy!*

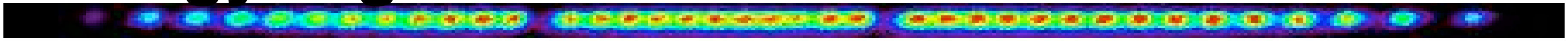
$$H\,|\psi_{E,n}\rangle = E\,|\psi_{E,n}\rangle$$

$$\Rightarrow \quad i\hbar\,\frac{d}{dt}\,|\psi_{E,n}\rangle = E\,|\psi_{E,n}\rangle$$

$$\boxed{\Rightarrow \quad \{|\psi_{E,n}\rangle(t)\} = e^{-iE_n t/\hbar}\,\{|\psi_{E,n}\rangle(t=0)\}}$$

- *linear vector space* ⇒ *solve other states by superposing solutions to energy eigenstates*

Energy Eigenstates:



aside: *how do we figure out energy eigenstates?*

- *usually amounts to solving spatial PDE:*

spatial distribution of quantum state $|\psi\rangle$

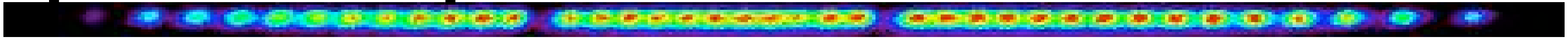
$(\psi(x) = \langle x|\psi\rangle)$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = (E - U)\,\psi(x)$$

total energy of quantum state $|\psi\rangle$

potential energy (gives the ***forces****)*

qubits vs. “qubits”:



- *so why do we care?!?…*

<u>qubits:</u>

- two-level quantum system
- basis states $|0\rangle$ and $|1\rangle$
- superpositions

$$\alpha|0\rangle + \beta|1\rangle$$

<u>“qubits” (real world):</u>

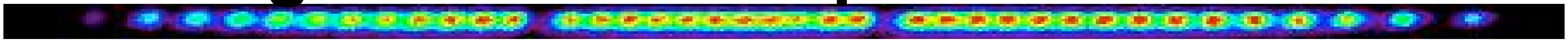
- in a non-empty universe, multi-level quantum systems
- superpositions

$$\alpha|0\rangle + e^{-iE_{01}t/\hbar}\beta|1\rangle$$

  - must track phases!

    ⇒ stable oscillator
- decoherence!

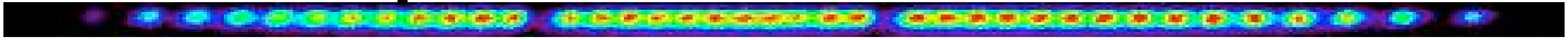
Building Quantum Computers:



To build a “real” quantum computer, need:

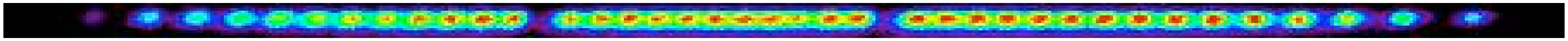
1. qubits
   - two-level quantum systems (or effective ones…)
   - *superpositions* □ isolated from outside world
   - confined, characterizable, scalable
2. preparation
   - prepare computer in standard start state
3. read-out
4. logic gates
   - controllable interactions with outside world!
   - single- and two-qubits gate sufficient (not nec.!)

Atoms as qubits

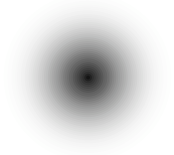


- why atoms?
  1. (my) familiarity…
  2. atoms are *the* standard for quantum superpositions
     - definition of the second:
       - phase evolution of superposition of atomic levels in Cesium
       - accuracy and stability ~ $1/10^{16}$

Quantum atoms:



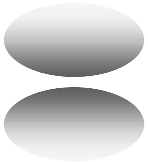
atoms: + charged nucleus, □ charged electrons



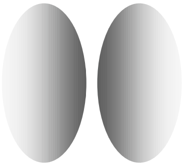
$(n, l, m_l) = (1,0,0)$

$(n, l, m_l) = (2,0,0)$

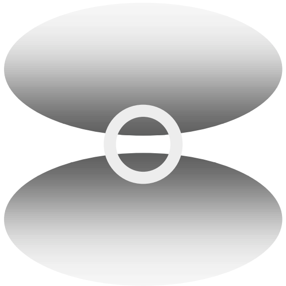
$(n, l, m_l) = (3,0,0)$



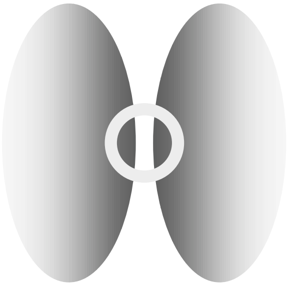
$(n, l, m_l) = (2,1,0)$



$(n, l, m_l) = (2,1,\pm 1)$



$(n, l, m_l) = (3,1,0)$

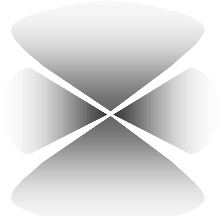


$(n, l, m_l) = (2,1,\pm 1)$

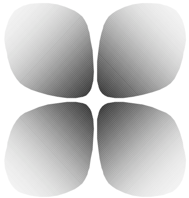
$n \sim h$

$l -$ a

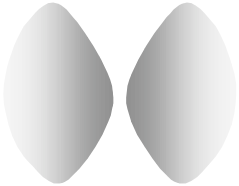
$m_l \sim$ prob. it's rotating about the direction you're looking…



$(n, l, m_l) = (3,2,0)$

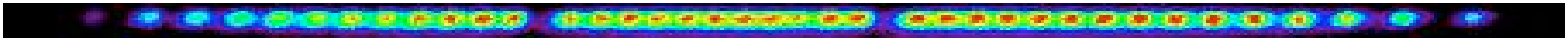


$(n, l, m_l) = (3,2,\pm 1)$

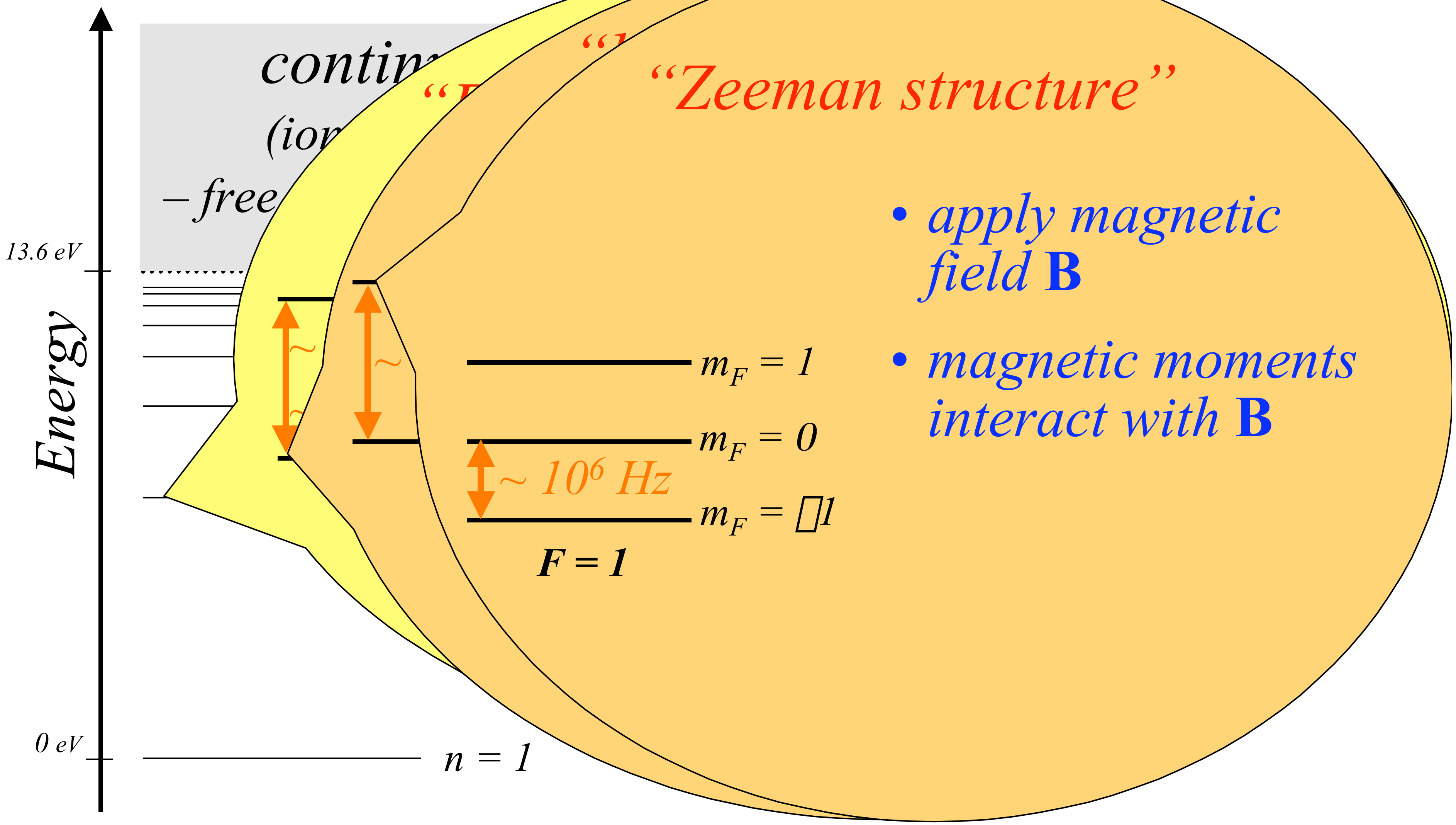


$(n, l, m_l) = (3,2,\pm 2)$

Atoms:

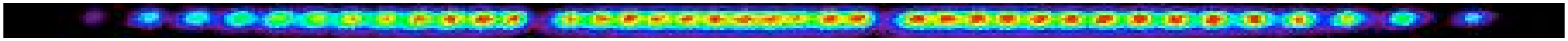


- energy-level diagram
  - e.g. atom with a singl hydrogen)



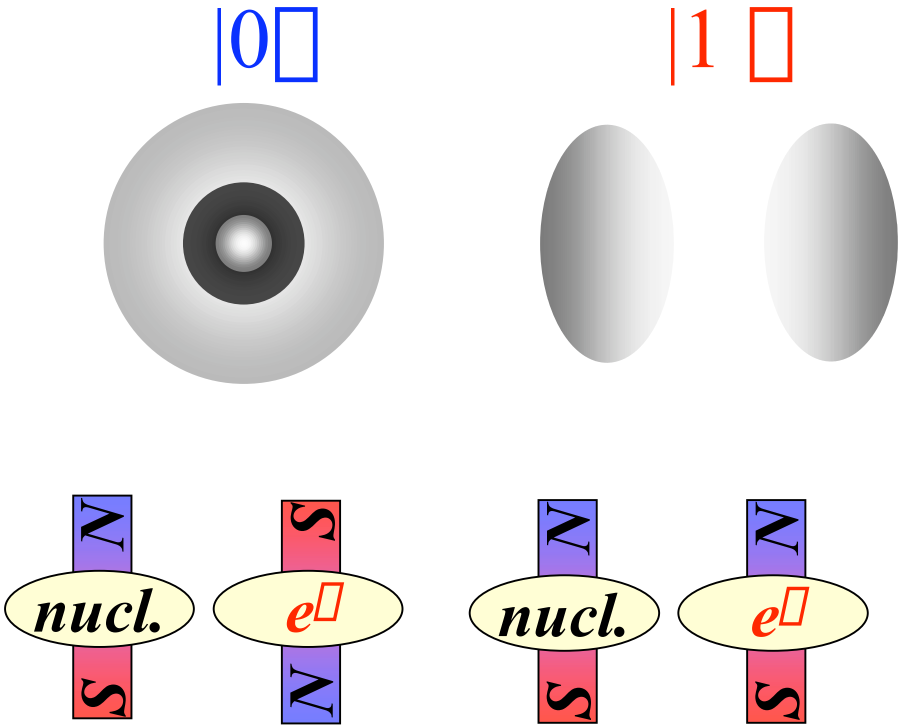
- *apply magnetic field* **B**
- *magnetic moments interact with* **B**

Atoms:



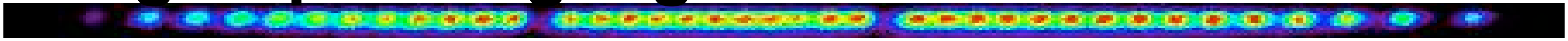
- **qubits:** isolate two energy levels
  - e.g. lowest-energy level (thermodynamics)

  \+ another level you can get to with some coupling

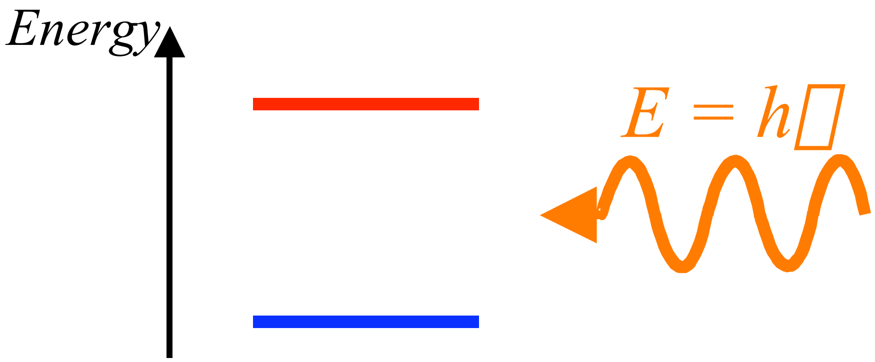


1. long-lived electronic levels (τ > *1 ms*)
2. long-lived ground-state hyperfine levels (τ > *10,000 y.*)

Single-qubit logic gates:



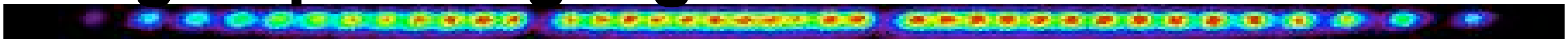
- analog of classical “*NOT*” gate - expanded!
- equivalent to preparation of arbitrary qubit state
- apply laser/microwaves!

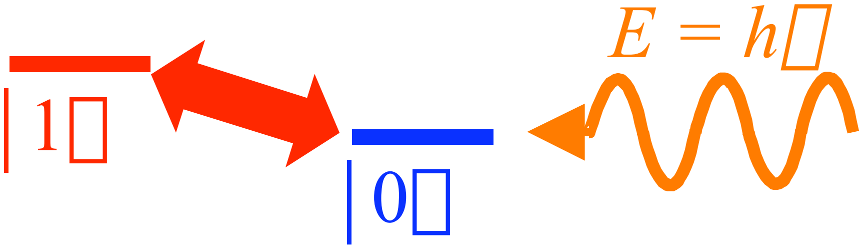


- absorption ↑
- stimulated emission □
- ~~spontaneous emission~~
  - (long-lived ex. state...)

- frequency must (approximately*) match the energy “gap” between the energy levels

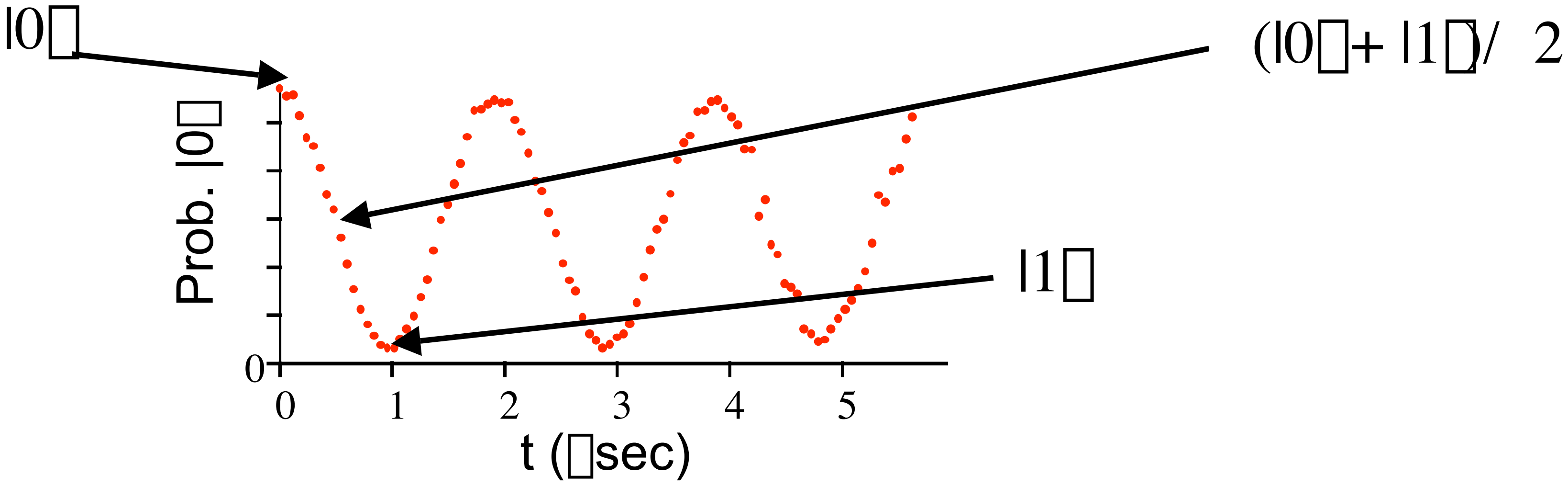
Single-qubit logic gate:



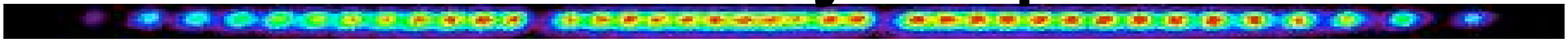


- classical: random hopping (either/or)
- quantum: flow of wave-function with time

$E_0 \neq E_1$

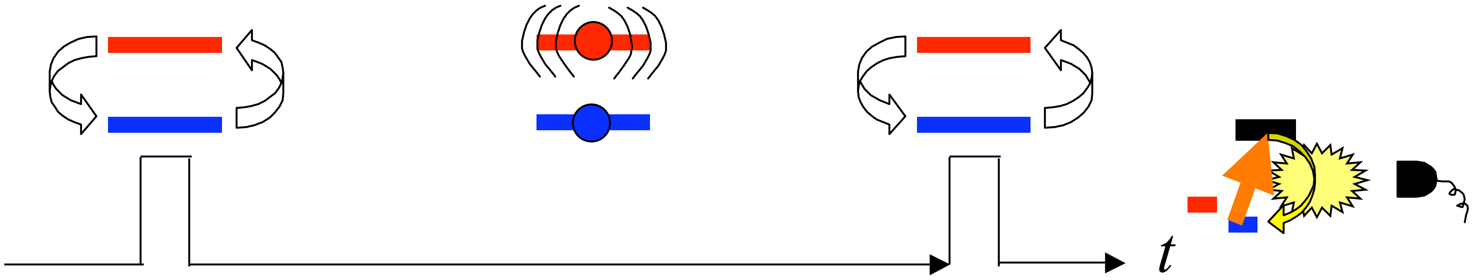


Nobel Sidebar - Ramsey’s expt.:



N. F. Ramsey, Rev. Mod. Phys. **62**, 541-552 (90)

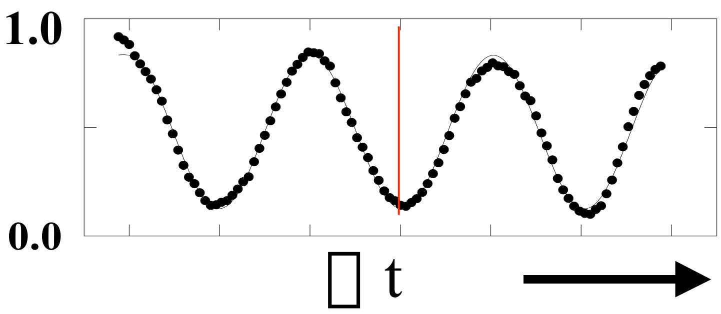
- **superpositions** - how do we characterize phase?



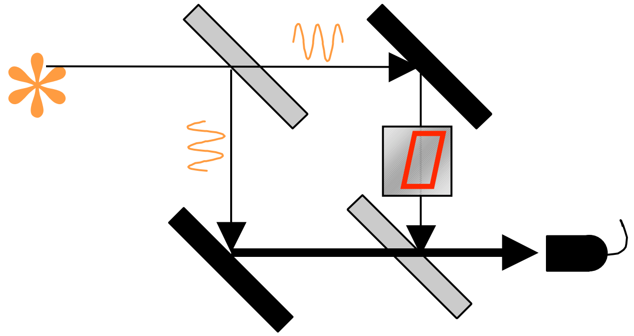
T/2:
create
superposition
~ Hadamard

t_R:
phase evolves
(Schrodinger)

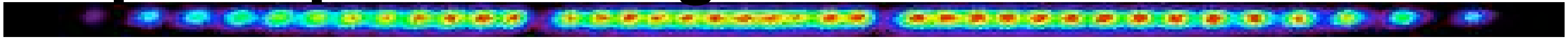
T/2, phase □:
try to undo
superposition!



- interferometer



2-qubit quantum logic – overview:



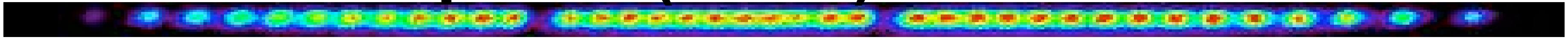
- 2-qubit gates ≡ conditional dynamics
  - e.g. “flip qubit #2 iff. qubit #1 = $|1\rangle$”
- need coupling between the qubits

  (also need coupling for entanglement…)

  e.g.

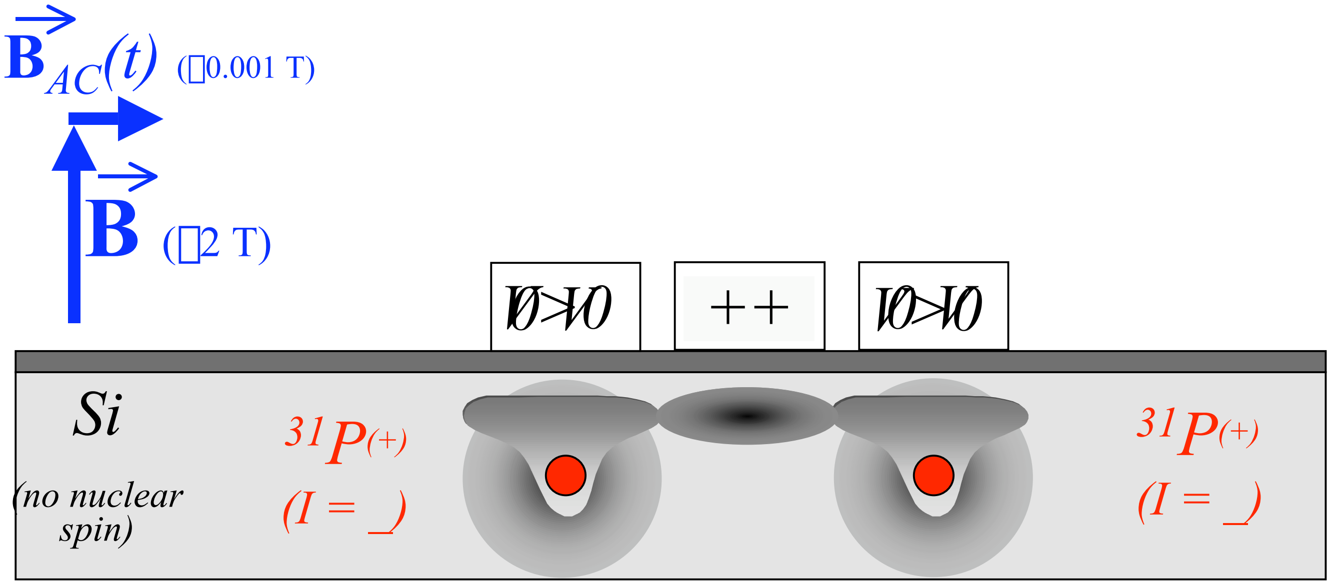
  - Coulomb interaction
  - dipole-dipole coupling
  - exchange interaction, Fermi exchange…
- must be able to (effectively) turn coupling ON/OFF!

Solid-state qubits (Kane)



B.E. Kane, Nature 393, 133 (98)

- nuclear spin qubits: **B** splits the energy of |□□, |↑□
  - energies modified by *e*□ distributions (hyperfine)
  - use chip technology to change *e*□ distributions



0 V □ E_{HF}

$\mathbf{B}_{AC}$ not resonant

V > 0 □ E_{HF}'

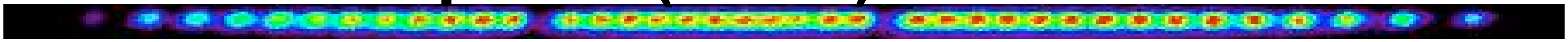
$\mathbf{B}_{AC}$ resonant

U > 0

e□ e□ interaction

□ nucl. interac.

Solid-state qubits (Kane)

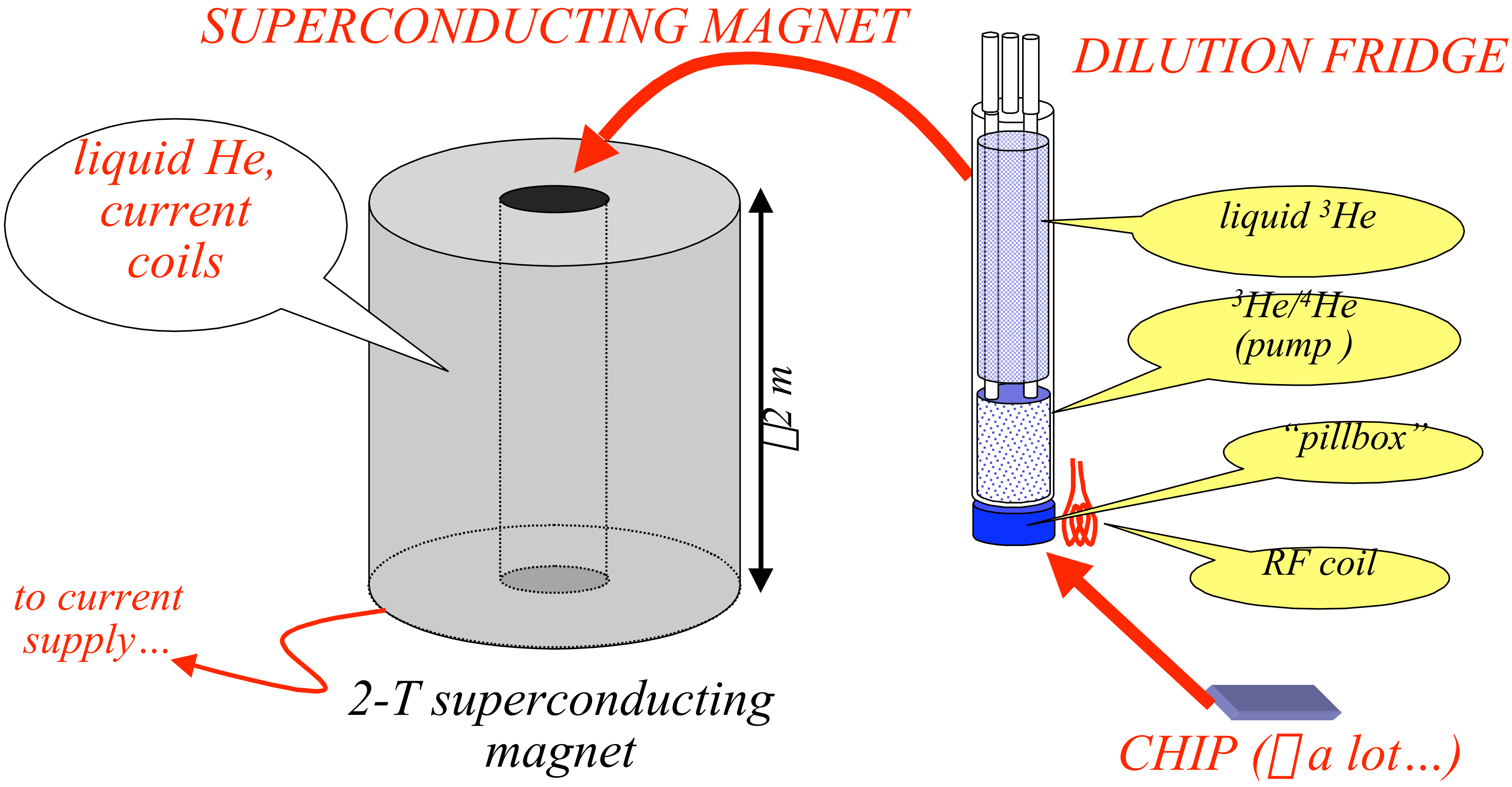


- initialization:
  - run at $k_B T \ll \mu_B B \Rightarrow T < 100\ mK, B > 2\ T$
- readout:
  - map nuclear spin onto e^- spin
  - apply gate voltage
  - only get current if e^- are in singlet (↑↓ − ↓↑)
- problems?
  - donor placement
  - decoherence mechanisms in bulk? (charge noise)
  - readout?

Kane, cont'd.:

- So what will it look like?

*…I have ***no*** idea… ("dammit, Jim, I'm an atomic physicist, not a condensed matter physicist!")



SUPERCONDUCTING MAGNET

DILUTION FRIDGE

liquid He, current coils

□2 m

liquid ^{3}He

^{3}He/^{4}He (pump)

"pillbox"

RF coil

to current supply…

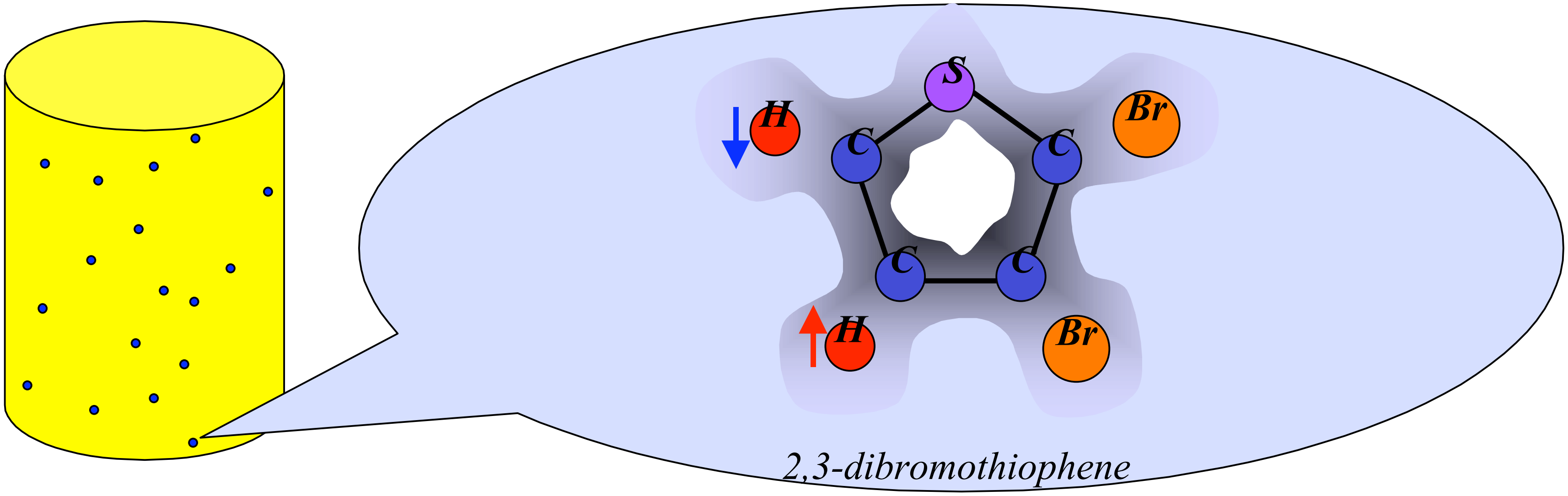
2-T superconducting magnet

CHIP (□a lot…)

Room-T, liquid-state NMR:

Cory, Fahmy, and Havel, Proc. Nat. Acad. Sci. USA **94**, 1634 (97); Gershenfeld and Chuagng, Science **275**, 350 (97); Jones and Mosca, J. Chem. Phys. **109**, 1648 (98), Laflamme, Knill, Zurek, Catasti, and Mariappan, Phil.Trans.Roy.Soc.Lond. **A356,** 1941 (98).

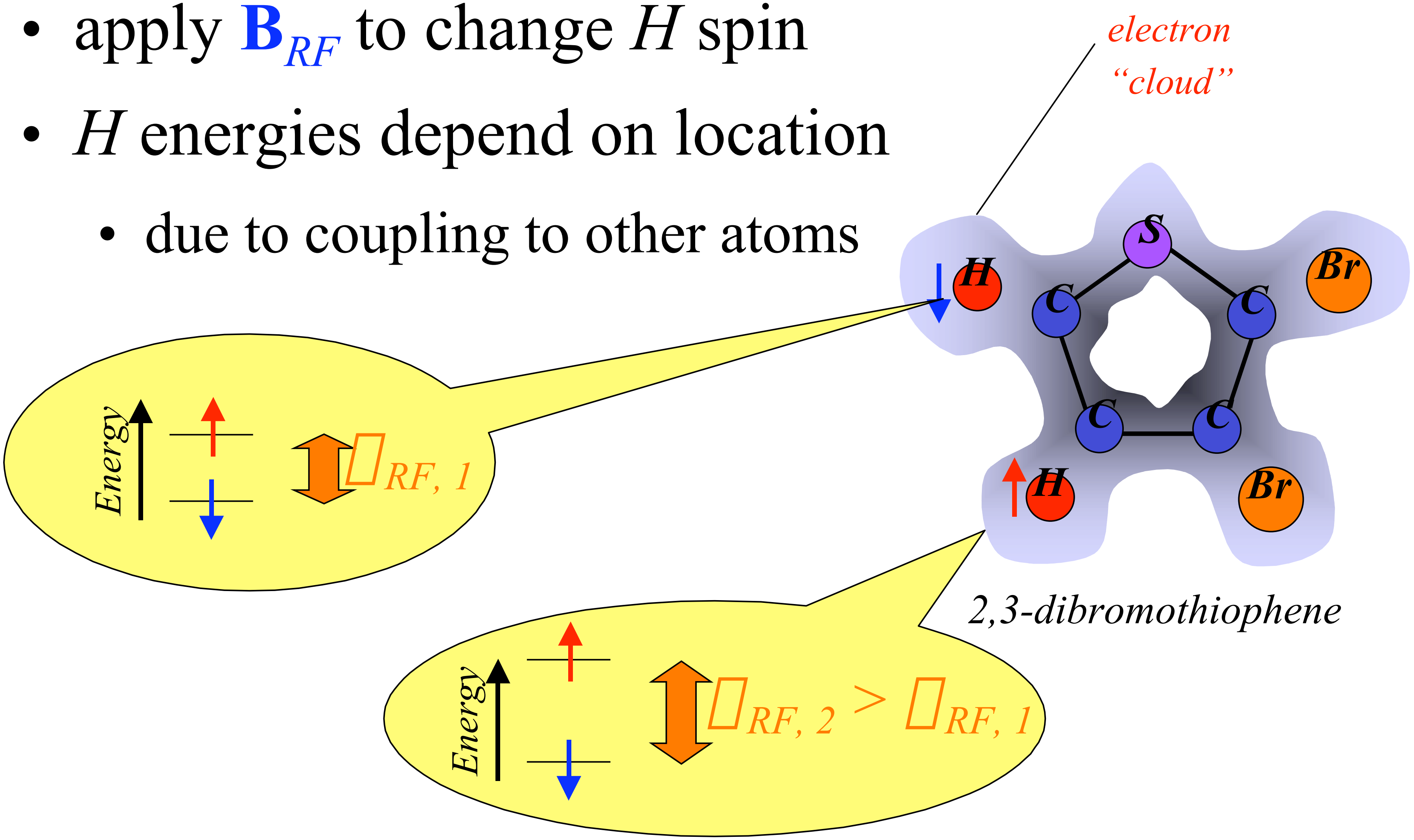
- ensemble of $N \sim 10^{22}$ molecules
  - room temperature
  - liquid (□ molecular interactions negligible…)
- each molecule has *n* spin-_´s (or higher)
  - e.g. hydrogen/protons, carbon, …
- *each molecule* is **one** quantum computer



2,3-dibromothiophene

NMR, cont'd.:

- apply **B** to split *H* spin energy levels
- apply $\mathbf{B}_{RF}$ to change *H* spin
- *H* energies depend on location
  - due to coupling to other atoms



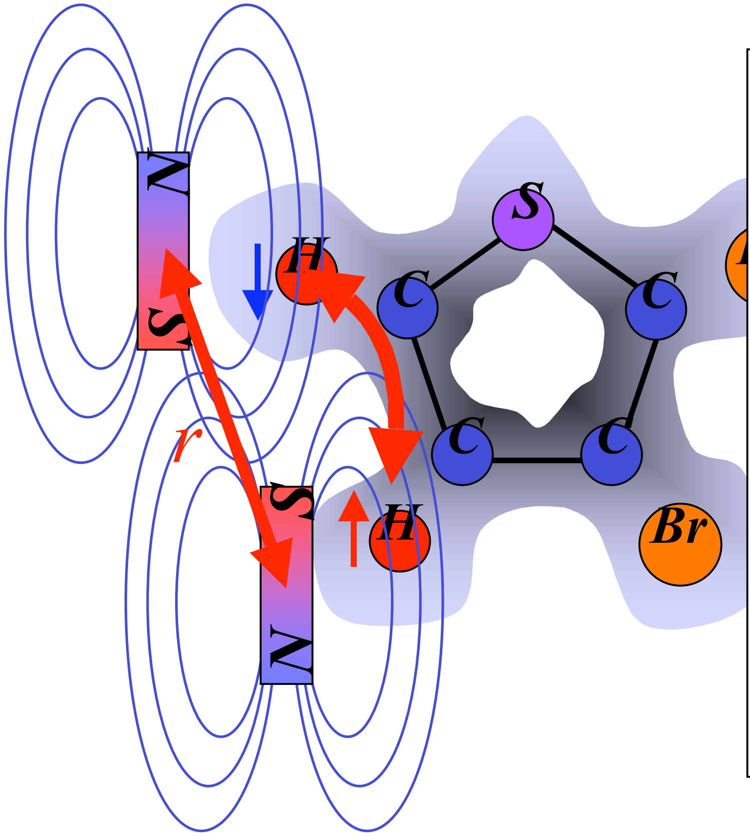
NMR, cont'd.:

- Coupling:

$$H_{int} = \frac{-\square_1\square_2}{4\, r^3}\,\underbrace{[\square_1\cdot\square_2 - 3(\square_1\cdot n)(\square_2\cdot n)]} + \frac{-J}{8}\,[\underbrace{\square_1\cdot\square_2}]$$

Dipolar coupling

•averages to zero if molecules are tumbling



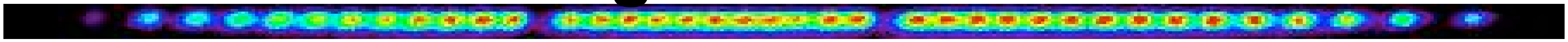
J-coupling

•molecular orbitals:

•atoms share electrons through chemical bonds

$\square J\, \square_{z1}\, \square_{z2}\, /4$

NMR Quantum logic:



- single-qubit gates:
  - apply $\mathbf{B}_{RF}$
- two-qubit gates:
  - *don't* do nothing!…
  - J-coupling is always on
  - *but*

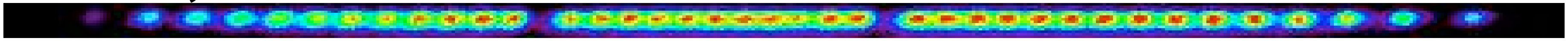
$$e^{\Box 2i\Box\Box_{x1}}\, e^{\Box\, ia\Box_{z1}\, t/\_}\, e^{\Box 2i\Box\Box_{x1}} = e^{+\, ia\Box_{z1}\, t/\_}$$

$$\Box \quad e^{\Box\, iJ\Box_{z1}\Box_{z2}\, t/4\_}\, e^{\Box 2i\Box\Box_{x1}}\, e^{\Box\, iJ\Box_{z1}\Box_{z2}\, t/4\_}\, e^{\Box 2i\Box\Box_{x1}} = e^{\Box 2i\Box\Box_{z2}\, t/\_}$$

"refocussing"

- turn off for quantum logic!

NMR, cont'd.:



- **"but NMR is done at room T!??!!…"**

1. "effective pure state"
   - thermodynamics □ less population in higher-energy states

     $\square\ \square\square\square\ \mathbb{1} + \square\, U|00\ldots0\rangle\langle 00\ldots0|U^\dagger$

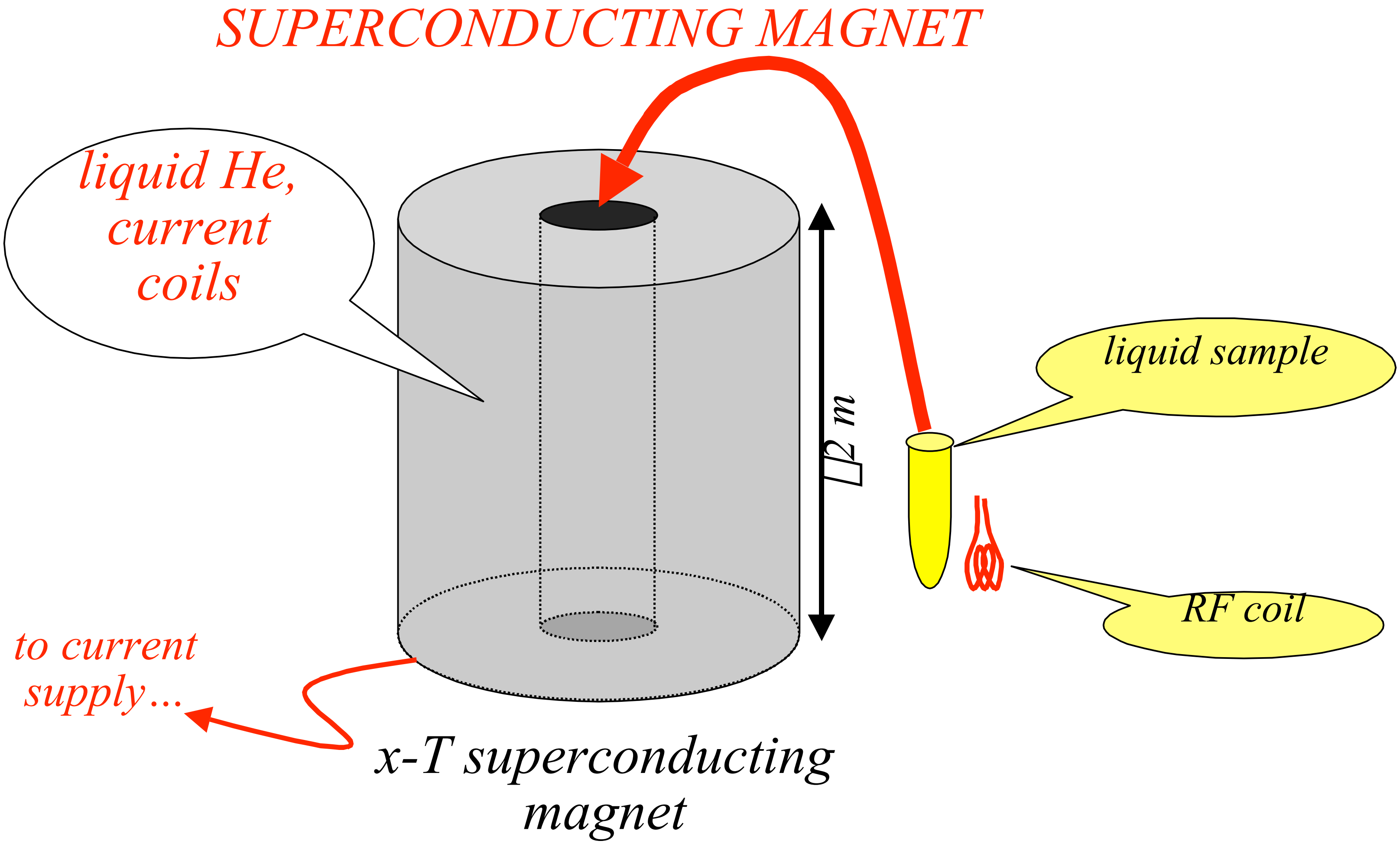
     (cost in resources…)
   - *readout insensitive to diagonal terms in □*

     □ *only read out* $|00\ldots0\rangle\langle 00\ldots0|$ part
2. ensemble readout
   - ensemble averages, but still works for some probs.

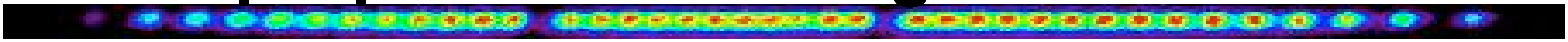
NMR, cont’d.:

- So what does it look like?

*…I have ***no*** idea… (dammit, Jim, I’m an atomic physicist, not a condensed matter physicist!)



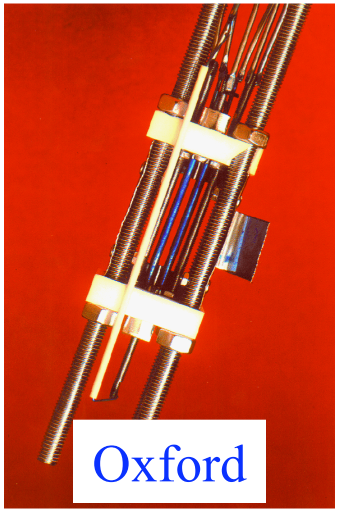
Other proposed technologies:

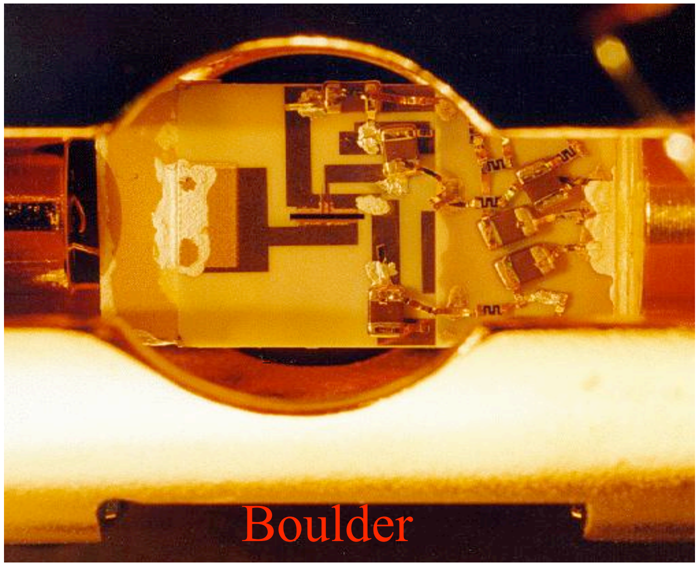


- <u>recall</u>
  - strong, switchable, controllable qubit interactions
  - no other interactions!
    - *photons* *(see also R. L.Fl.!)*
    - ?← Josephson junctions (Martinis, et al., Phys. Rev. Lett. 89, 117901 (02) and references therein…[Nakamura, Devoret, Martinis, Han,…]
    - ?← electron dots (Bayer *et al.*, Science **291**, 451 (03) and references therein…[Steel, Sherwin, Hawrylak,…])
    - electrons floating above liquid He
    - etc…
  - **trapped atomic ions**

Ion traps for quantum computing:

- store quantum information inside atoms
  - build on existing quality of superpositions
  - need way to implement 2-qubit gates
  - plus a lot more!…


Oxford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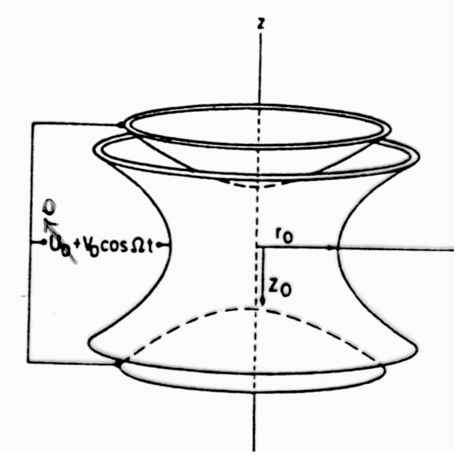
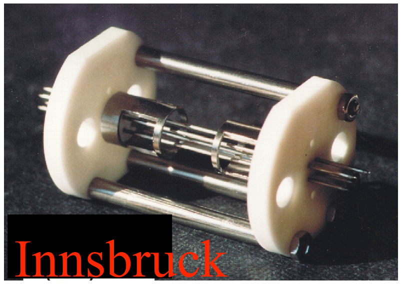


G. Werth, *Progress in Atomic Spectroscopy*,
H.J. Beyer, H. Kleinpoppen, *eds*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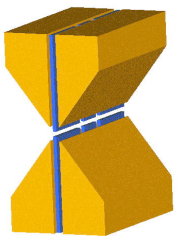
MPQ/Garching