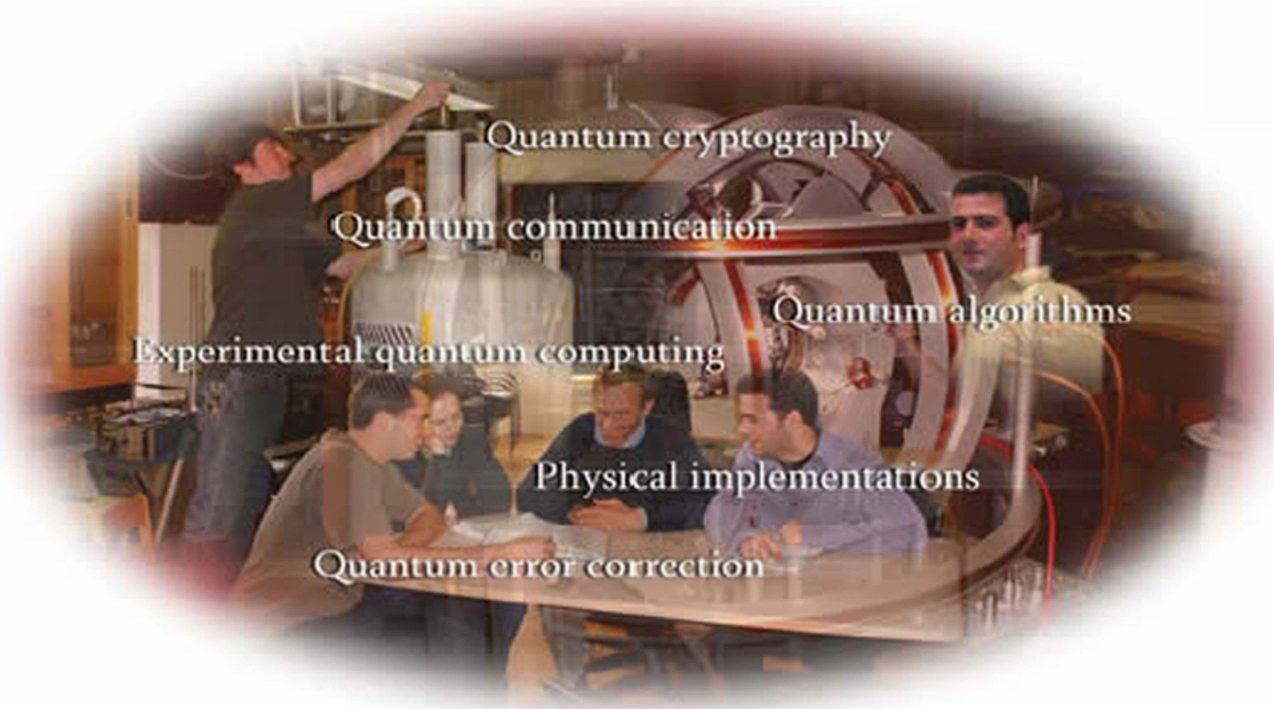
IQC Institute *for* Quantum Computing



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Linear Optics Quantum Computing

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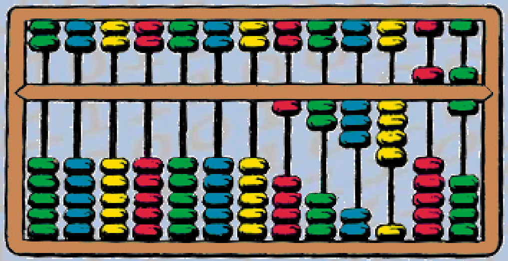
www.perimeterinstitute.ca

Plan

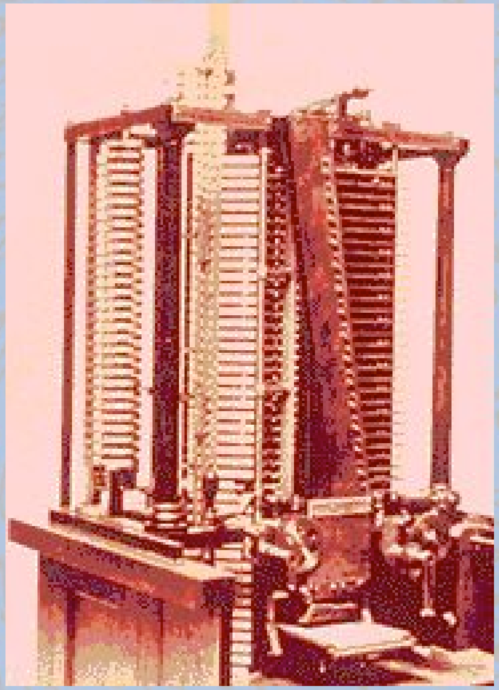
- Introduction: history and context for LOQC
  • devices physics, • what is Q.Info., • QEC
- Quantum Teleportation
- Linear Optics Quantum Computation
- Quantum error correction and threshold
- Conclusion

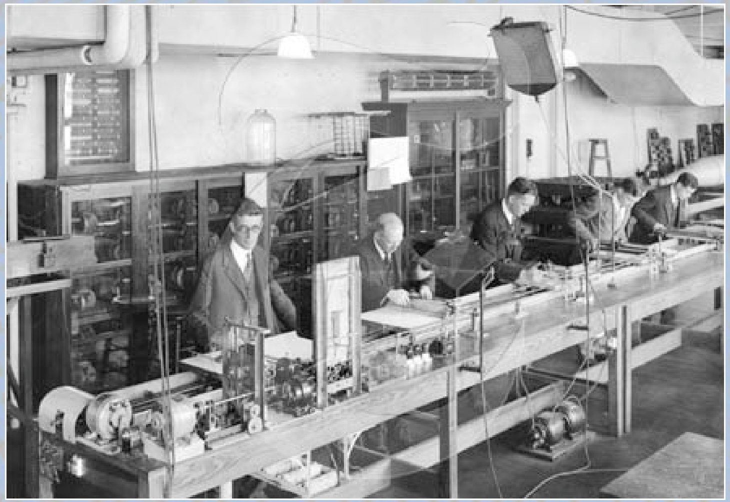
with Manny Knill, Gerard Milburn, Hamed Majedi and Casey Myers (Nature 409, 46-52, 2001).

Computing devices



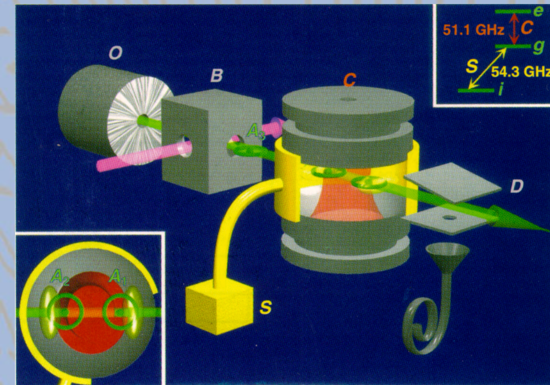
Babage's analytical engine



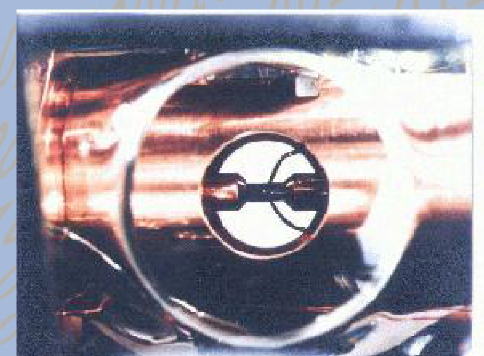


MIT's mechanical mind

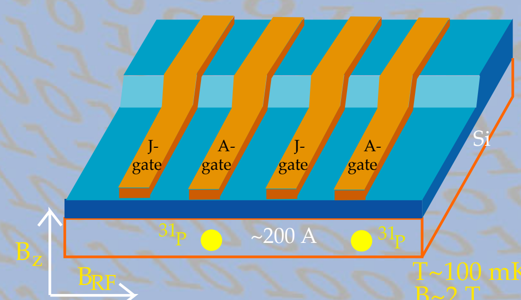
Devices for Quantum Information Processing

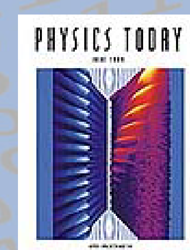


- Atom traps
- Cavity QED
- Electron floating on helium
- Electron trapped by surface acoustic waves
- Ion traps



- Nuclear Magnetic Resonance
- Quantum Optics
- Quantum dots
- Solid state





- Spintronics
- Superconductivity

Requirements for QC (DiVincenzo)

- **Hilbert and subsystems**
- **Initial state**
- **Control**
- **Measurement**
- **Low noise, decoherence (error correction)**

Note: efficiency in resources is critical

Threshold theorem

A quantum computation can be as long as required with any desired accuracy as long as the noise level is below a threshold value

$$P < 10^{-4}$$

Knill et al.; Science, 279, 342, 1998

Kitaev, Russ. Math Survey 1997

Aharonov & Ben Or, ACM press

Preskill, PRSL, 454, 257, 1998

Significance:

-imperfections and imprecisions are not fundamental objections to quantum computation

-it gives criteria for scalability

-its requirements are a guide for experimentalists

-it is a benchmark to compare different technologies

Harmonic oscillator and the EM field

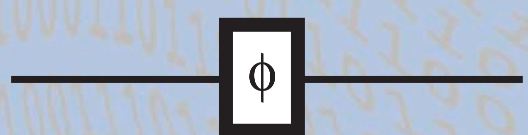
$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 x^2$$

$$\hat{a} = \frac{1}{\sqrt{2}} \left(\sqrt{\frac{m\omega}{\hbar}} \hat{x} + i \frac{1}{\sqrt{m\hbar\omega}} \hat{p} \right)$$

$$a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle$$
$$a |n\rangle = \sqrt{n} |n-1\rangle$$

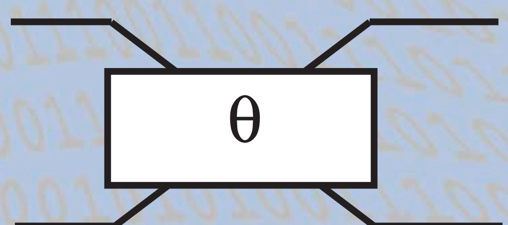
$$a = \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{2} & 0 & \cdots \\ 0 & 0 & 0 & \sqrt{3} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Phase shifter on 1st qubit:



$$a_1^\dagger \rightarrow e^{-i\phi} a_1^\dagger$$

Beam splitter



$$\begin{pmatrix} a_1^{\dagger\prime} \\ a_2^{\dagger\prime} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_1^\dagger \\ a_2^\dagger \end{pmatrix}$$

Optical Quantum Computation

- quantum optics well developed
- photons are cheap
- room temperature
- long coherence time

- need non-linearity
- inefficient photon source/detector
- scalability

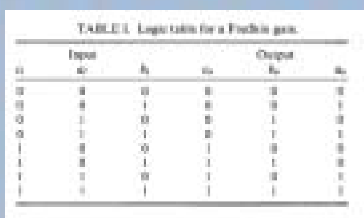
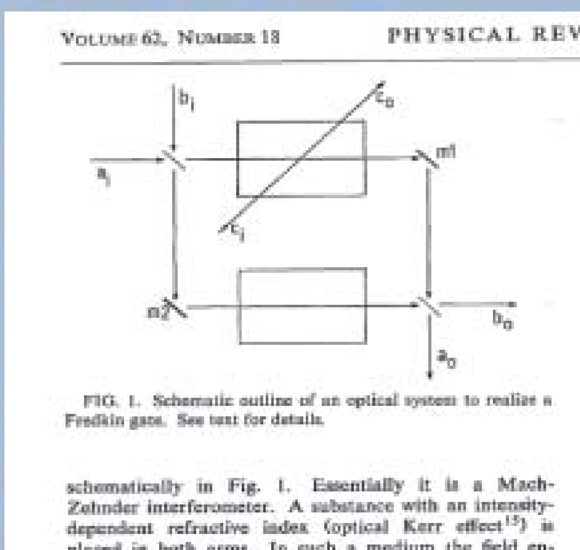


TABLE I. Logic table for a Fredkin gate.

Input			Output		
c_i	a_i	b_i	c_o	b_o	a_o
0	0	0	0	0	0
0	0	1	0	0	1
0	1	0	0	1	0
0	1	1	0	1	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	0	1
1	1	1	1	1	1

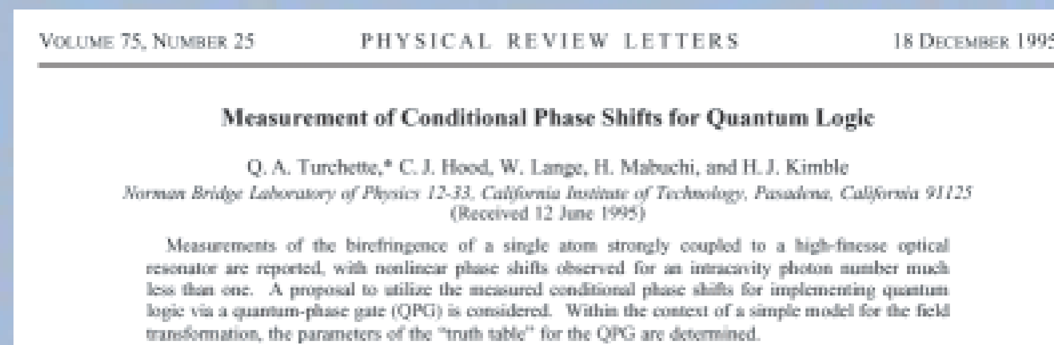
Milburn,
PRL62,2124, 1989



VOLUME 62, NUMBER 18 PHYSICAL REV

FIG. 1. Schematic outline of an optical system to realize a Fredkin gate. See text for details.

schematically in Fig. 1. Essentially it is a Mach-Zehnder interferometer. A substance with an intensity-dependent refractive index (optical Kerr effect[15]) is placed in both arms. In such a medium the field en-

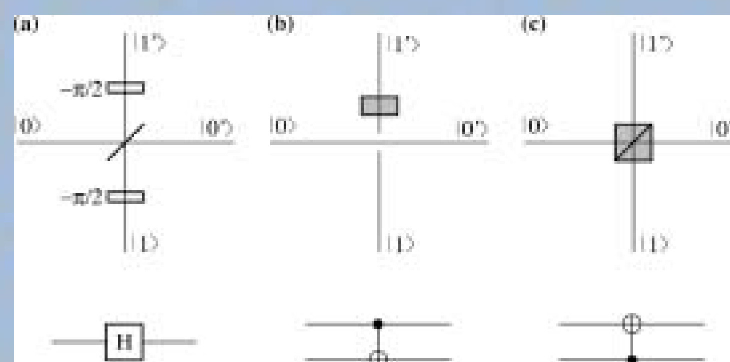


VOLUME 75, NUMBER 25 PHYSICAL REVIEW LETTERS 18 DECEMBER 1995

Measurement of Conditional Phase Shifts for Quantum Logic

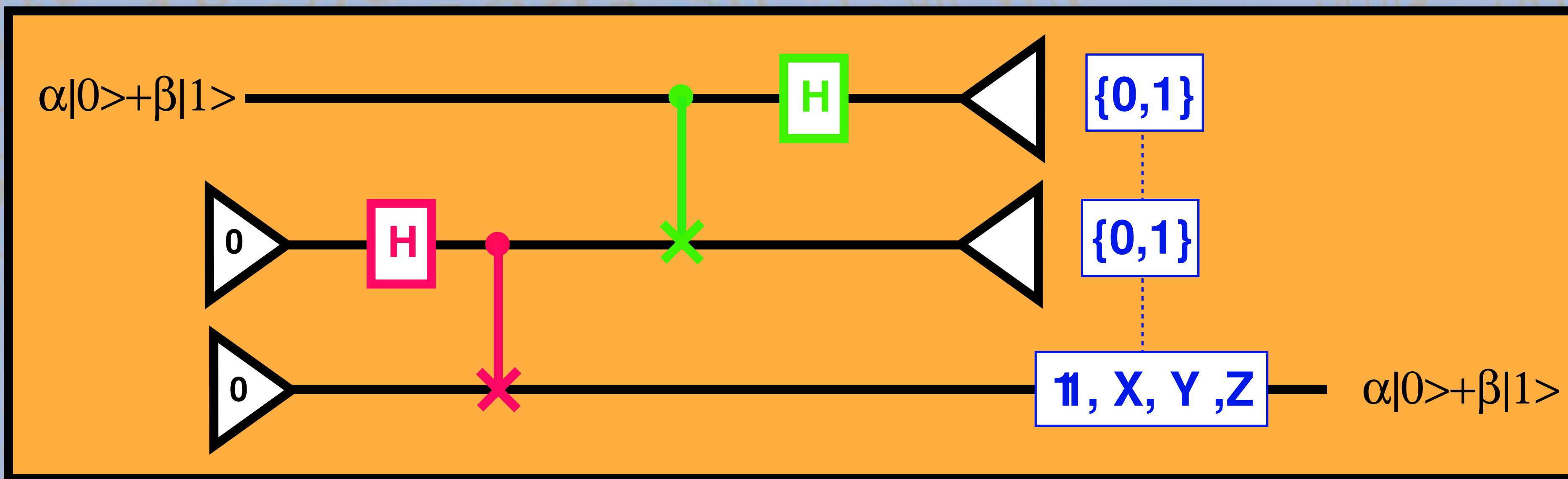
Q. A. Turchette,* C. J. Hood, W. Lange, H. Mabuchi, and H. J. Kimble
Norman Bridge Laboratory of Physics 12-33, California Institute of Technology, Pasadena, California 91125
(Received 12 June 1995)

Measurements of the birefringence of a single atom strongly coupled to a high-finesse optical resonator are reported, with nonlinear phase shifts observed for an intracavity photon number much less than one. A proposal to utilize the measured conditional phase shifts for implementing quantum logic via a quantum-phase gate (QPG) is considered. Within the context of a simple model for the field transformation, the parameters of the "truth table" for the QPG are determined.

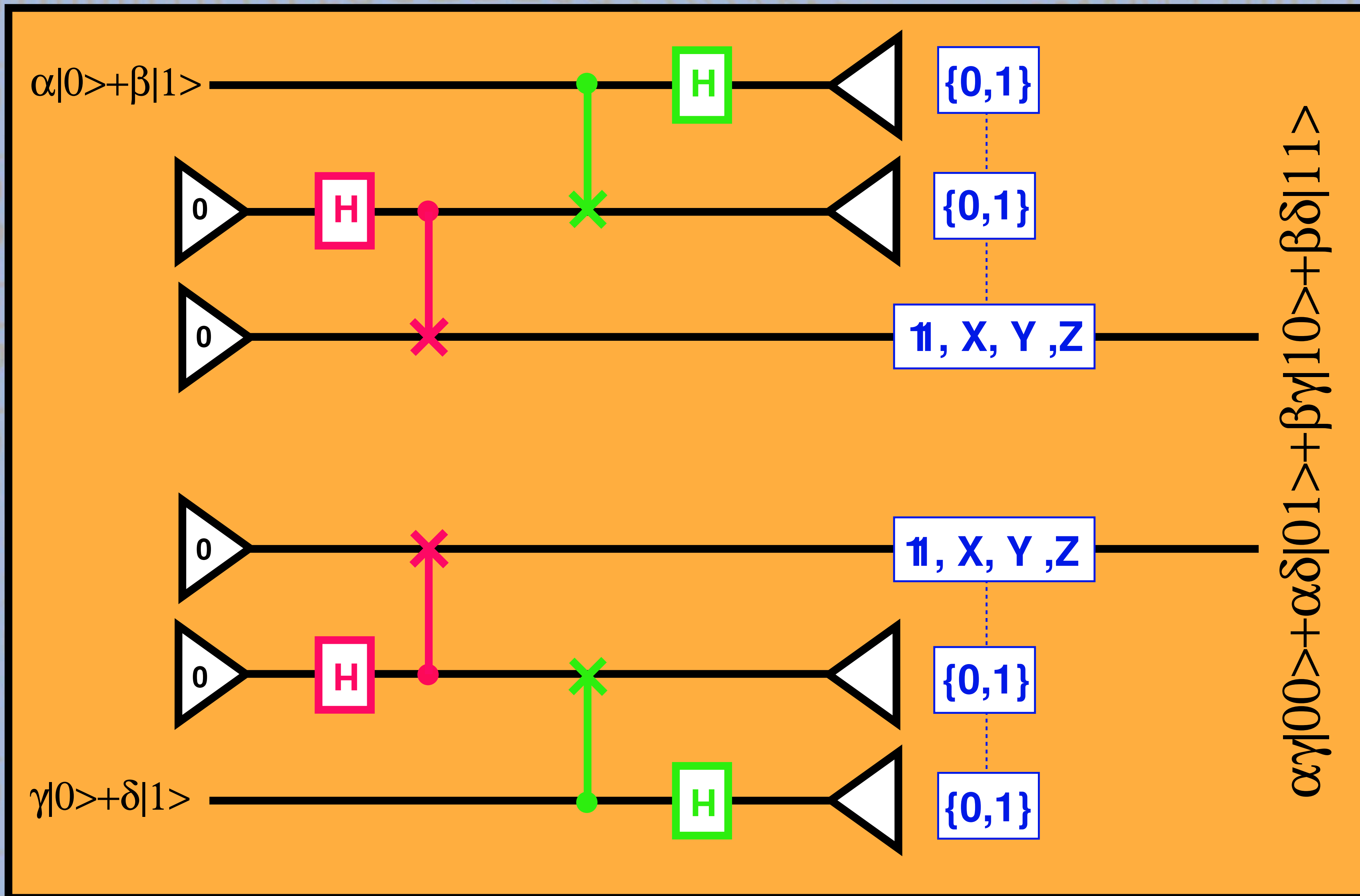


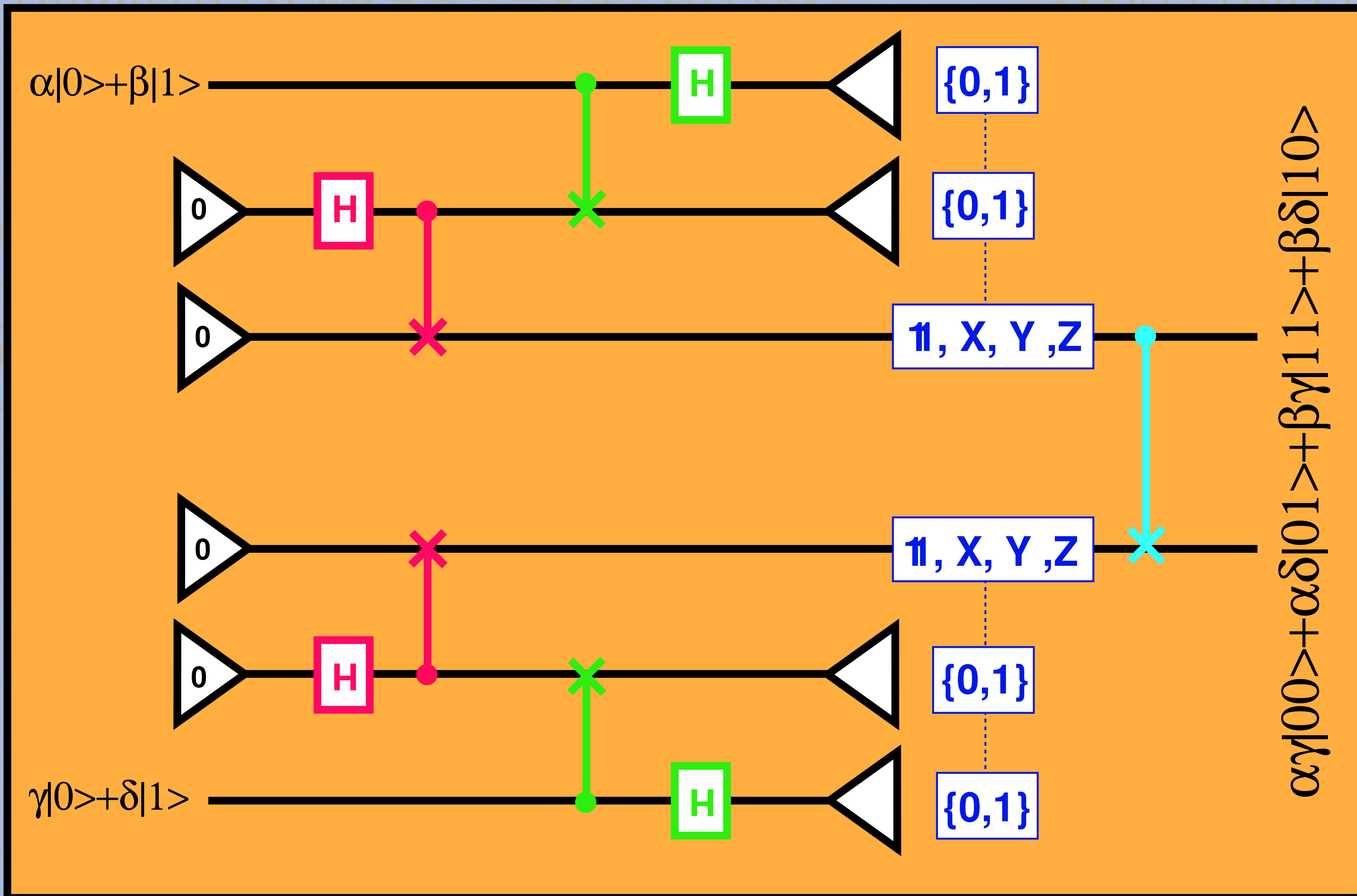
Cerf, Adami and Kwiat
PRA57, R1477, 1998

Quantum Teleportation

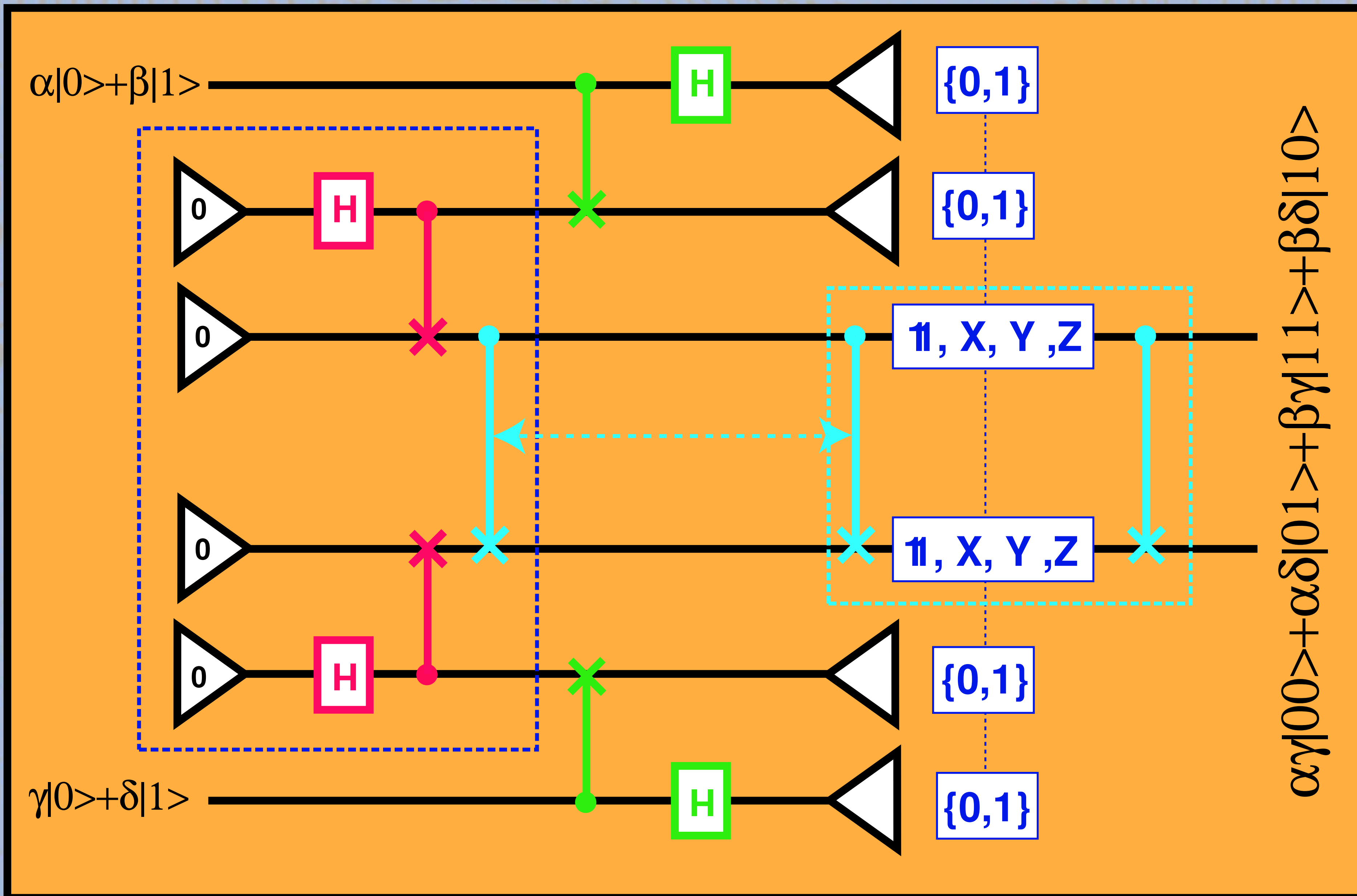


Bennett et al. PRL 70, 1895, 1993


α|0>+β|1>
γ|0>+δ|1>
0
0
0
0
H
H
H
H
{0,1}
{0,1}
{0,1}
{0,1}
𝟙, X, Y ,Z
𝟙, X, Y ,Z
αγ|00>+αδ|01>+βγ|10>+βδ|11>



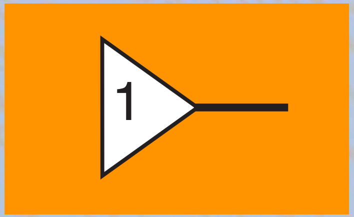
α|0>+β|1>
H
{0,1}
0
H
{0,1}
0
11, X, Y ,Z
0
11, X, Y ,Z
0
H
{0,1}
γ|0>+δ|1>
H
{0,1}
αγ|00>+αδ|01>+βγ|11>+βδ|10>



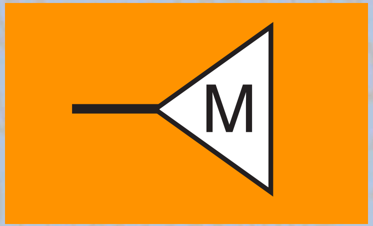
Gottesman and Chuang, Nature 402, 390, 1999

LOQC: one bit gates

Single photon source:



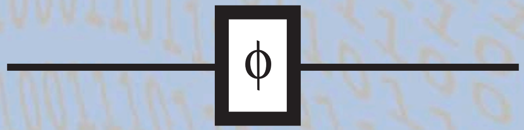
Single photon detector:



Qubit:

$$\alpha\underbrace{|01\rangle}_{|0_Q\rangle}+\beta\underbrace{|10\rangle}_{|1_Q\rangle} = \alpha a_2^\dagger + \beta a_1^\dagger |00\rangle$$

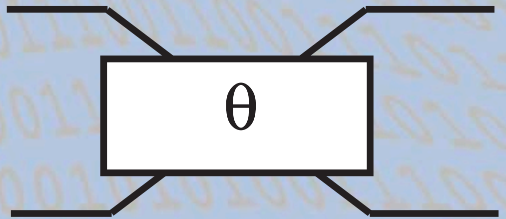
Phase shifter on 1st qubit:



$$a_1^\dagger \rightarrow e^{-i\phi} a_1^\dagger$$

$$|Q\rangle \rightarrow e^{-i\phi/2} e^{i\phi Z/2} |Q\rangle$$

Beam splitter



$$\begin{pmatrix} a_1^{\dagger\prime} \\ a_2^{\dagger\prime} \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_1^\dagger \\ a_2^\dagger \end{pmatrix}$$

$$|Q\rangle \rightarrow e^{i\theta Y} |Q\rangle$$

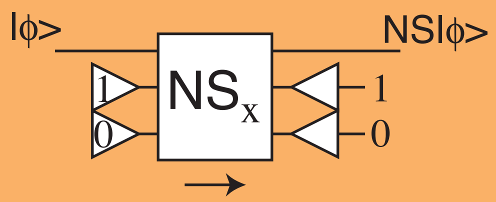
Rules: use only beam splitters, phase shifters
single photon source and detectors (detect 0,1,2 photons)

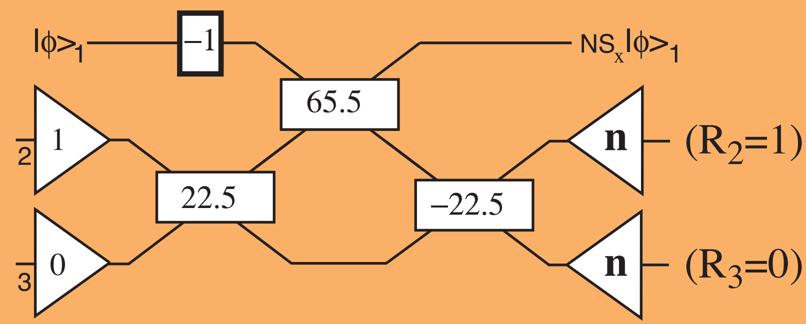
- Show a probabilistic gate which takes
  $a_0 |0\rangle + a_1 |1\rangle + a_2 |2\rangle \;\; \Rightarrow a_0 |0\rangle + a_1 |1\rangle - a_2 |2\rangle$
  from this you get a probabilistic CSIGN gate
- Use a simple adaption of quantum teleportation (probabilistic) to turn the difficulty of making a gate into the difficulty of making a state
- Improve the quantum teleportation scheme
  ($P_f = 1/(n+1)$)
- Use error correction to better control the failures

2 bit gates

Sign-Flip on a 2 photon mode: P=1/4

$|\phi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle$





$NS|\phi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle - \alpha_2|2\rangle$

$$(a + b * a_1^\dagger + c * (a_1^\dagger)^2) a_2^\dagger$$

$$\begin{pmatrix} a_2^{\dagger\prime} \\ a_3^{\dagger\prime} \end{pmatrix} = \begin{pmatrix} \cos 22.5 & -\sin 22.5 \\ \sin 22.5 & \cos 22.5 \end{pmatrix} \begin{pmatrix} a_2^\dagger \\ a_3^\dagger \end{pmatrix}$$

$$\begin{pmatrix} a_1^{\dagger\prime} \\ a_2^{\dagger\prime} \end{pmatrix} = \begin{pmatrix} \cos 65.5 & -\sin 65.5 \\ \sin 65.5 & \cos 65.5 \end{pmatrix} \begin{pmatrix} a_1^\dagger \\ a_2^\dagger \end{pmatrix}$$

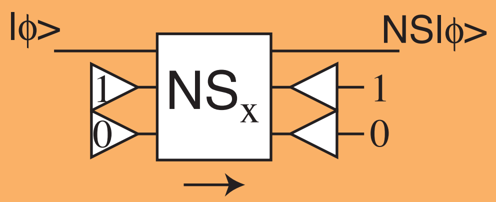
$$\begin{pmatrix} a_2^{\dagger\prime} \\ a_3^{\dagger\prime} \end{pmatrix} = \begin{pmatrix} \cos 22.5 & \sin 22.5 \\ -\sin 22.5 & \cos 22.5 \end{pmatrix} \begin{pmatrix} a_2^\dagger \\ a_3^\dagger \end{pmatrix}$$

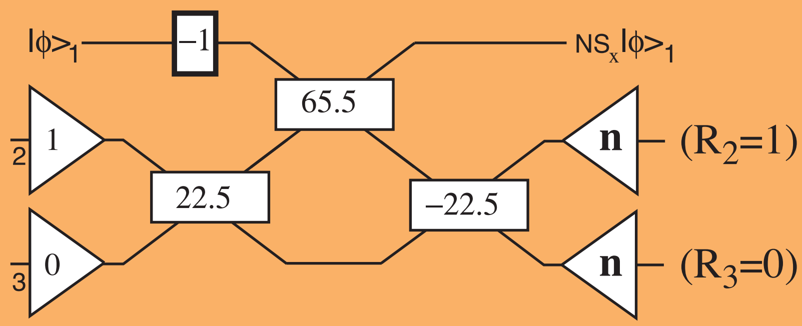
$$\frac{1}{2}(a + b * a_1^\dagger - c * (a_1^\dagger)^2) a_2^\dagger$$

2 bit gates

Sign-Flip on a 2 photon mode: P=1/4

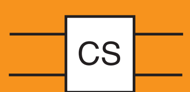
$|\phi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle + \alpha_2|2\rangle$



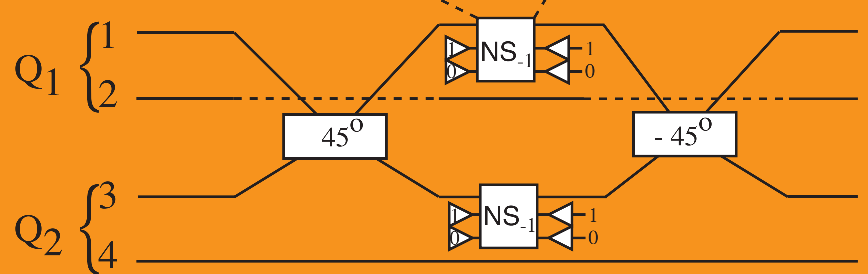


$NS|\phi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle - \alpha_2|2\rangle$

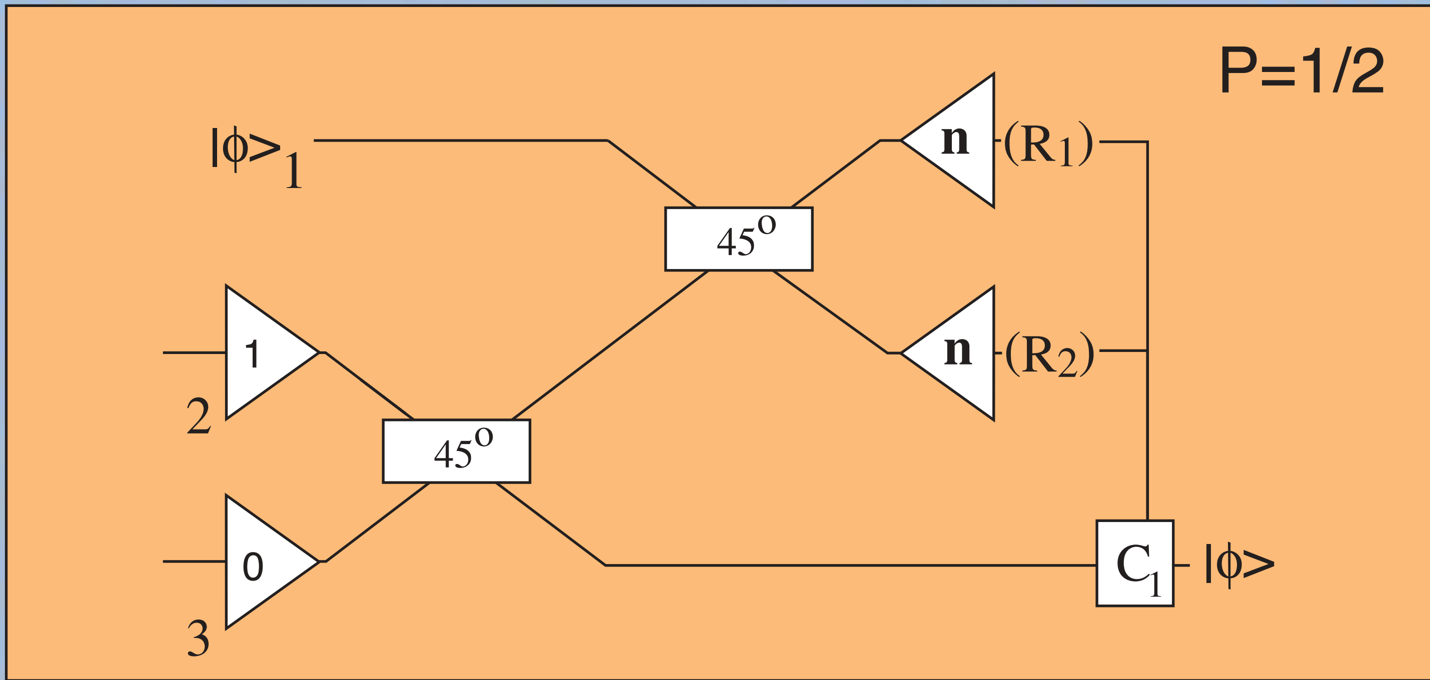
Control-Sign on Qubits: P=1/16



$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$



Probabilistic Quantum Teleportation



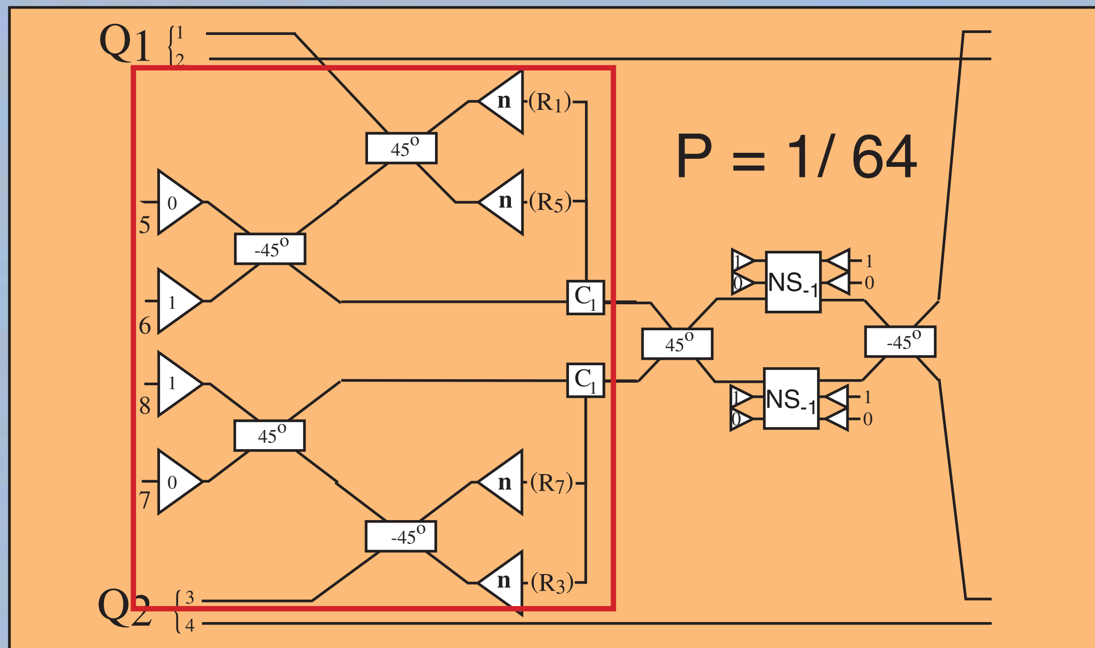
$$(\alpha|0\rangle + \beta|1\rangle)|10\rangle \quad \rightarrow \quad (\alpha|0\rangle + \beta|1\rangle)(|10\rangle - |01\rangle)/\sqrt{2}$$

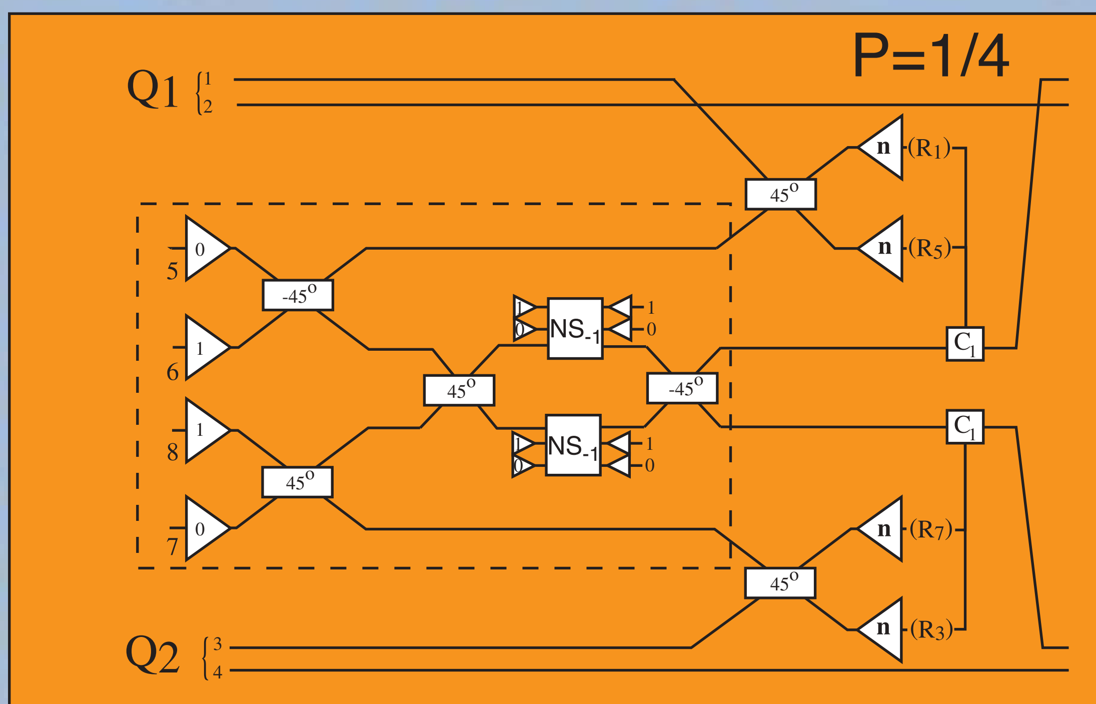
$$\rightarrow \frac{1}{2}|01\rangle(\alpha|0\rangle + \beta|1\rangle) + \frac{1}{2}|10\rangle(\alpha|0\rangle - \beta|1\rangle) +$$

$$\frac{1}{\sqrt{2}}|00\rangle\alpha|1\rangle + \frac{1}{2}(|20\rangle - |02\rangle)|\rangle\beta|0\rangle$$

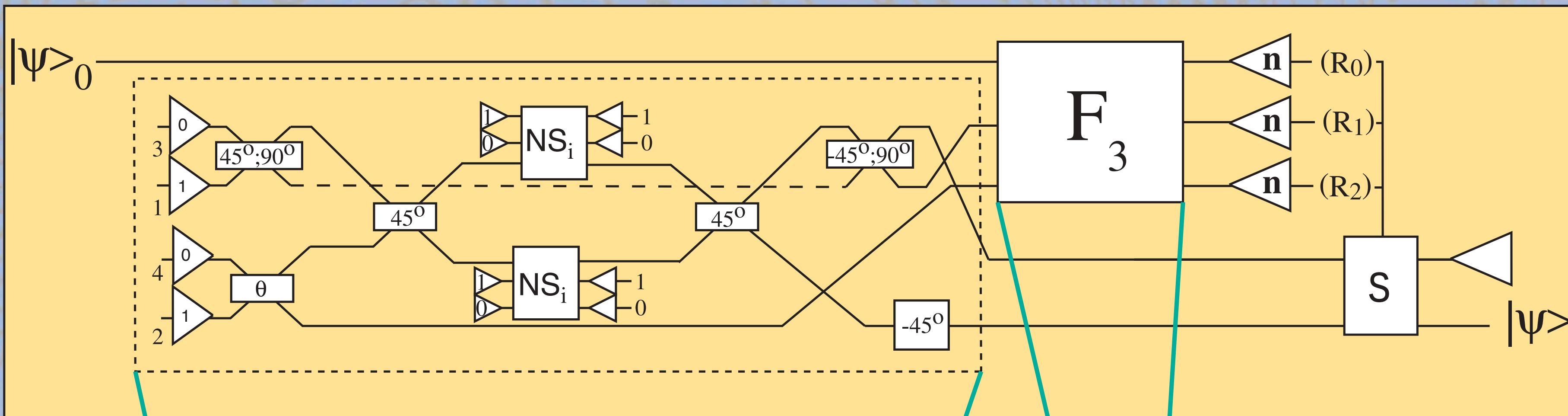
If R1+R2 =1 mod2, Quantum Teleportation succeed,
if R1=0 and R2=1, C=1 if not C=Z
If R1+R2 =0 mod2, Quantum Teleportation fails
Failure mode is a measurement of the initial state.

Control-Sign using Quantum Teleportation





Boosting the probability of success of the Control-Sign



$$|t_n\rangle = \sum_{j=0}^{n} |1\rangle^j |0\rangle^{n-j} |0\rangle^j |1\rangle^{n-j}$$

$$(F_n)_{kl} = \frac{\omega^{kl}}{\sqrt{n+1}}$$

$$\omega^{kl} = e^{\frac{i2\pi}{n+1}}$$

$$P_f = \frac{1}{n+1}$$

LOQC summary

-With single photon source/detector, phase shifter and beam splitter, we showed how to efficiently simulate a quantum computer.

-How can improve on what we have?

-Many sources of errors:

*error due to probabilistic gates
*imperfect sources/detectors
*mode absorption/dispersion
*imperfect beam splitters
*...

Correcting for Z basis measurement

-A critical element was the use of probabilistic quantum teleporation with an error model corresponding to a measurement in the Z (computational) basis.

-Maybe we can improve the proposal by using quantum error correction

Correcting for Z basis measurement

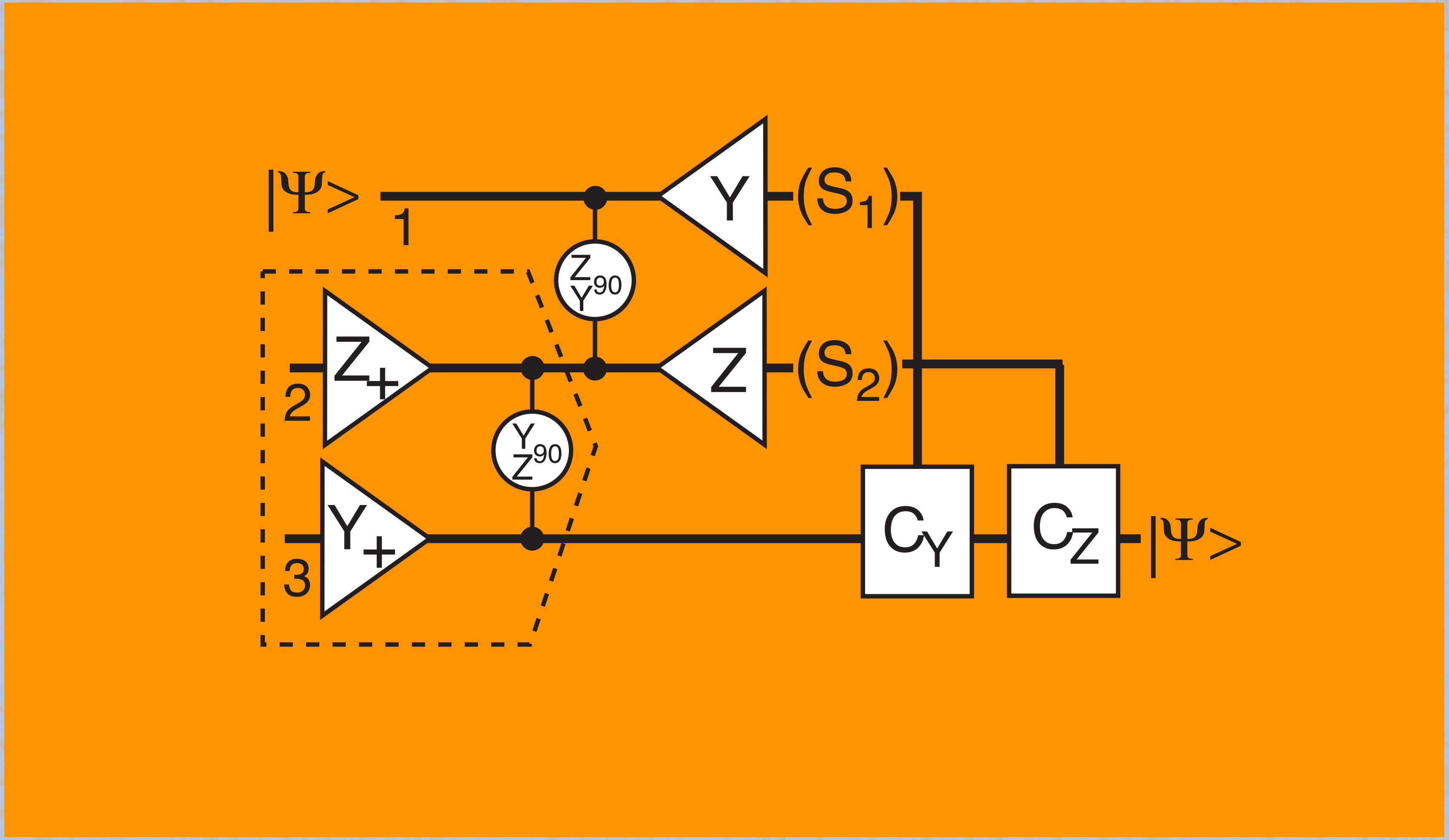
We have all one bit gates. The two bit gate however fail under projection in the Z basis and we know which qubit has been measured. This happens with probability f.

Can we error correct for that?

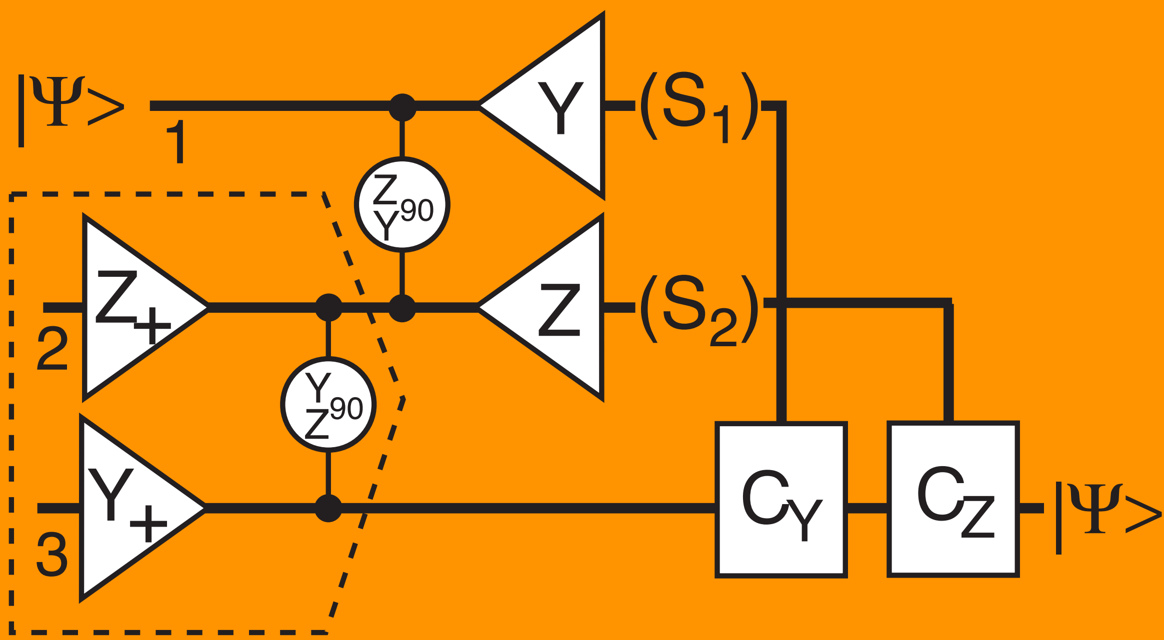
Yes, use the code:

$$\square_2: \quad \begin{aligned} |0\rangle &\rightarrow |00\rangle + |11\rangle \\ |1\rangle &\rightarrow |01\rangle + |10\rangle \end{aligned}$$

Nice Quantum Teleportation



Nice Quantum Teleportation



$$S_1 = -\; S_2 = - \rightarrow C_Y = \mathbb{1}\,, C_Z = \mathbb{1}$$

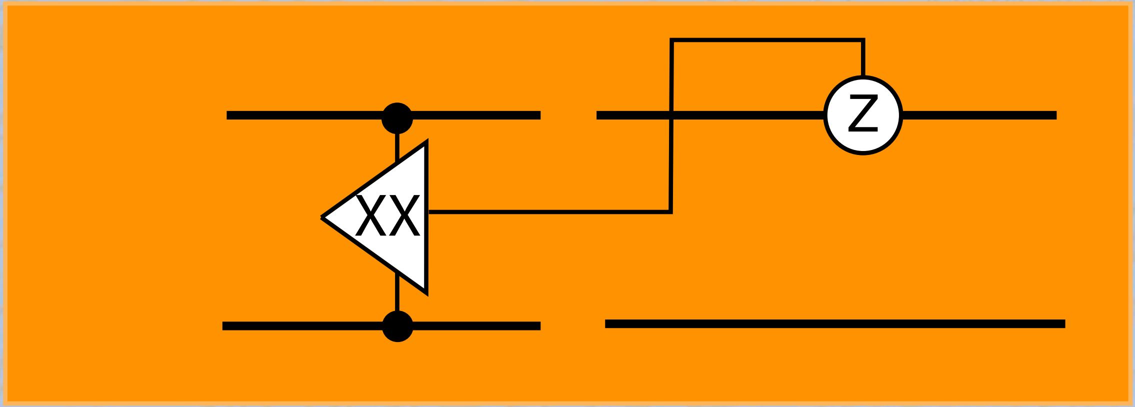
$$S_1 = +\; S_2 = - \rightarrow C_Y = Z\,, C_Z = \mathbb{1}$$

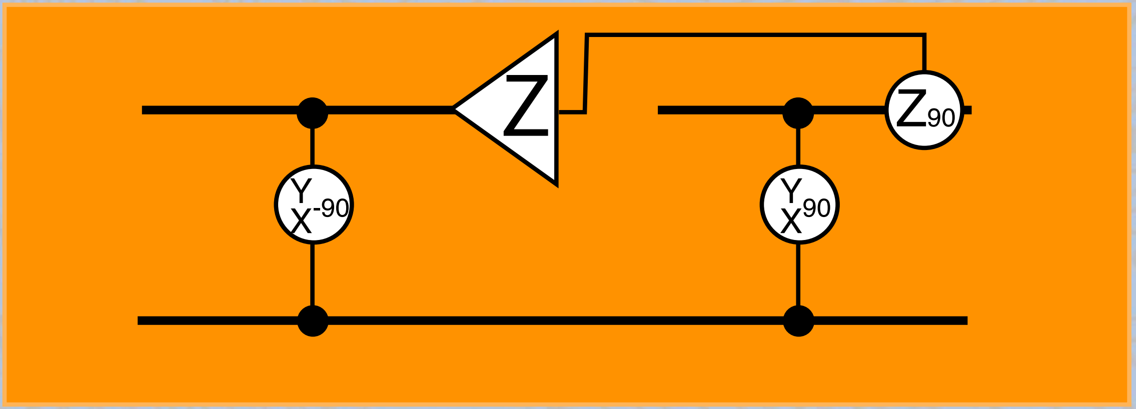
$$S_1 = -\; S_2 = + \rightarrow C_Y = \mathbb{1}\,, C_Z = X$$

$$S_1 = +,\; S_2 = + \rightarrow C_Y = Z\,, C_Z = X$$

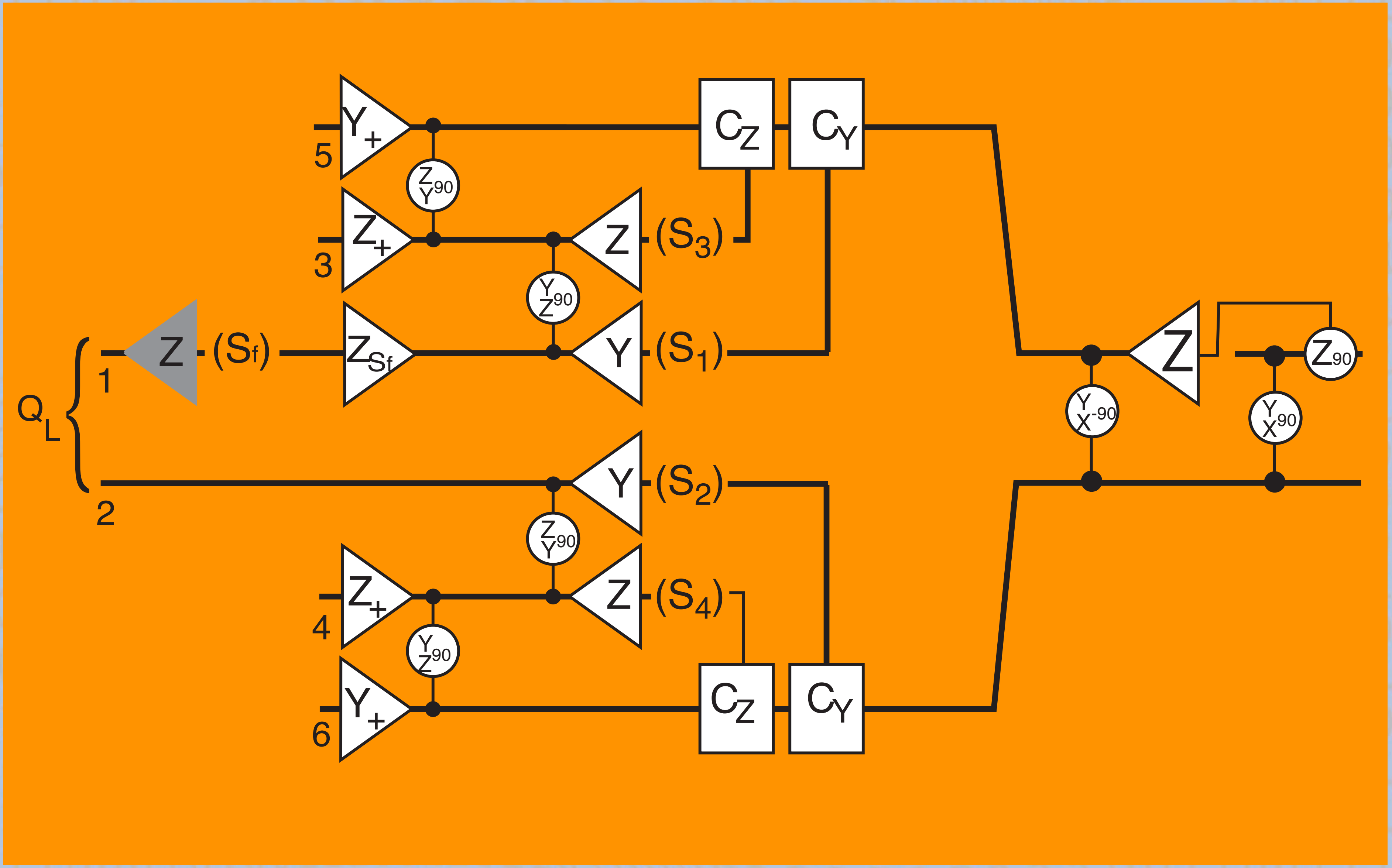
Error recovery

Projection in the XX basis

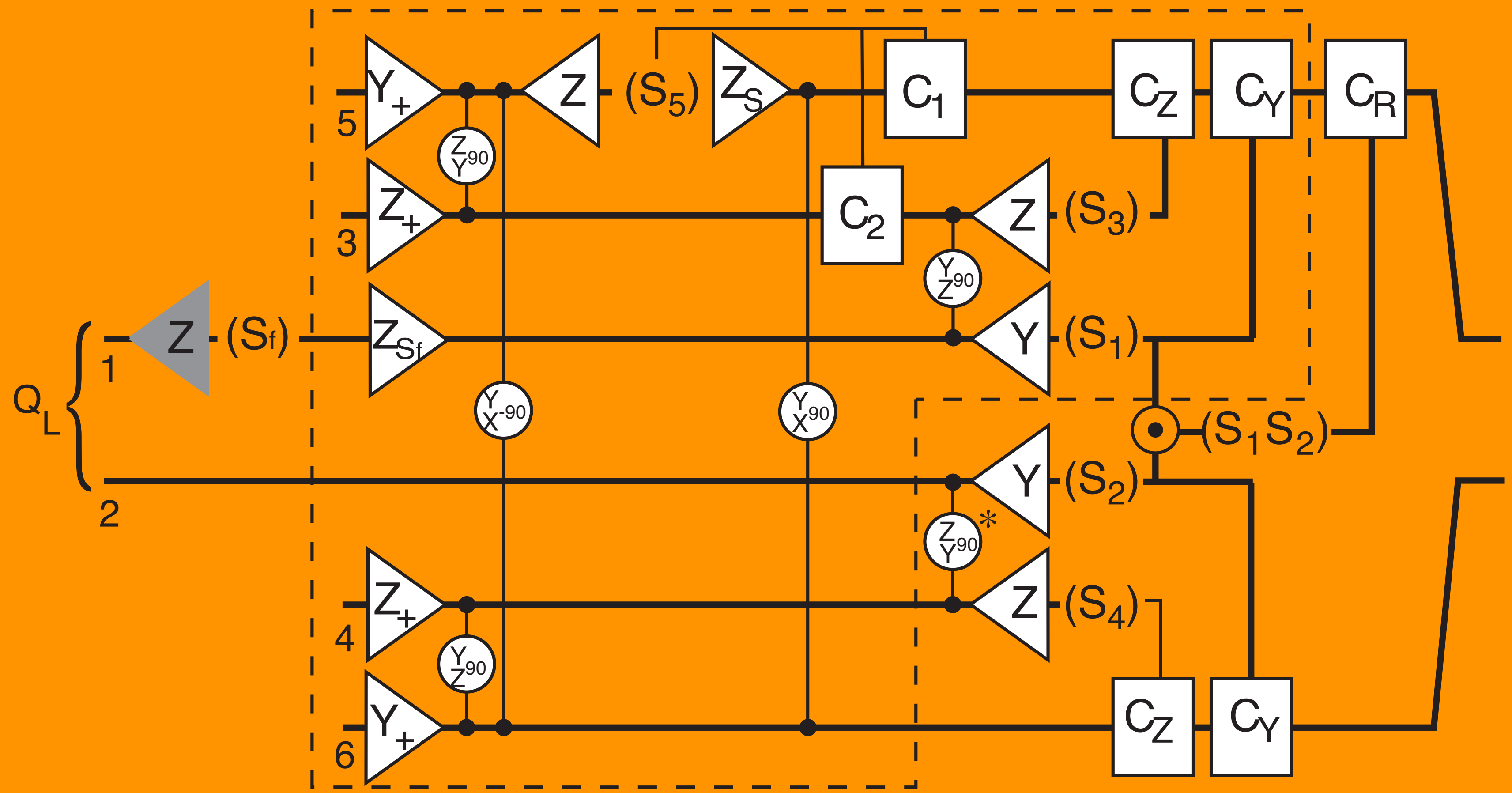




Error recovery



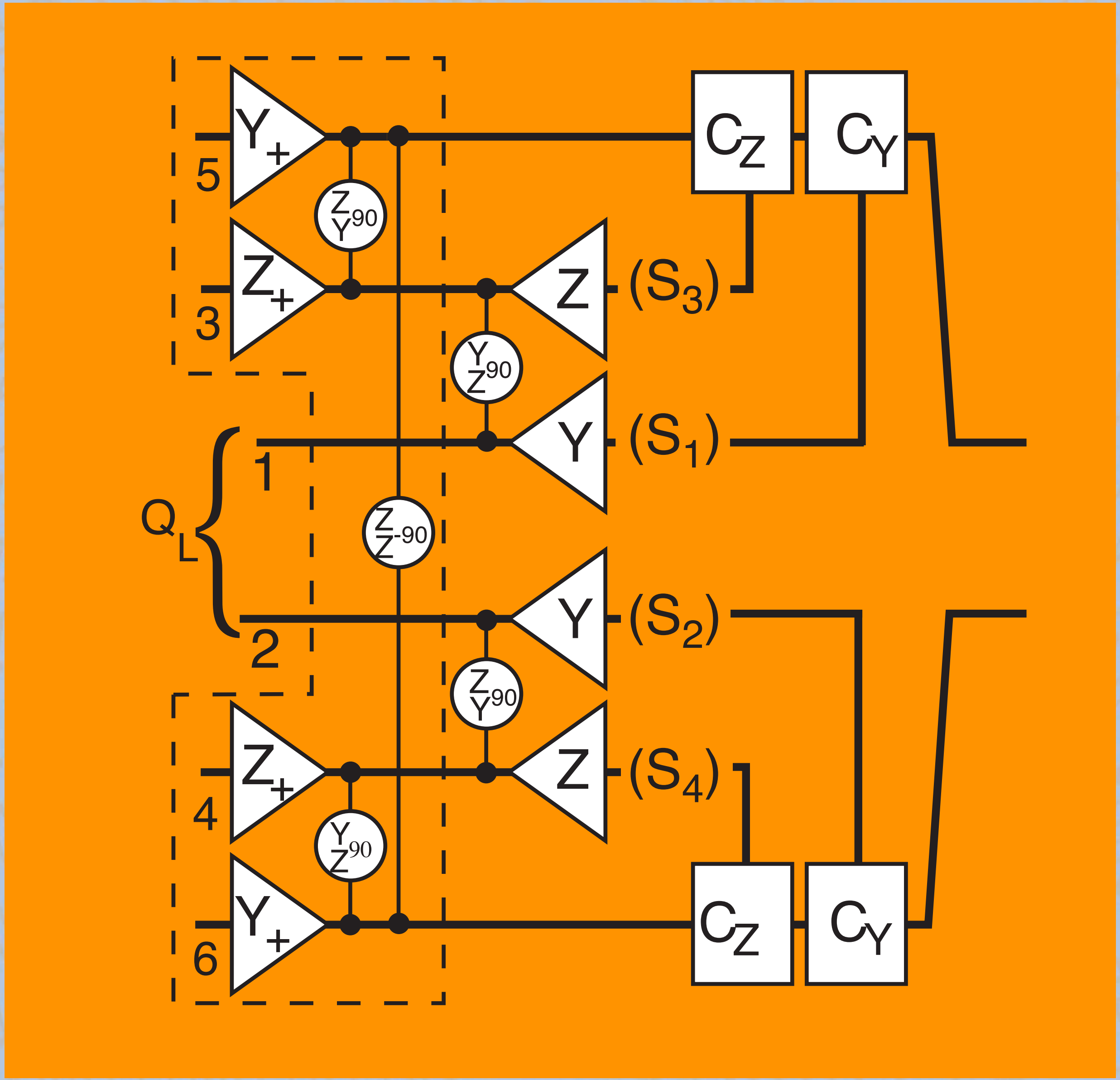
Error recovery



if control = - : C_1=Z C_2=X C_R=Z

$$F_r = f * F_r + (1-f)f \rightarrow F_r = f$$

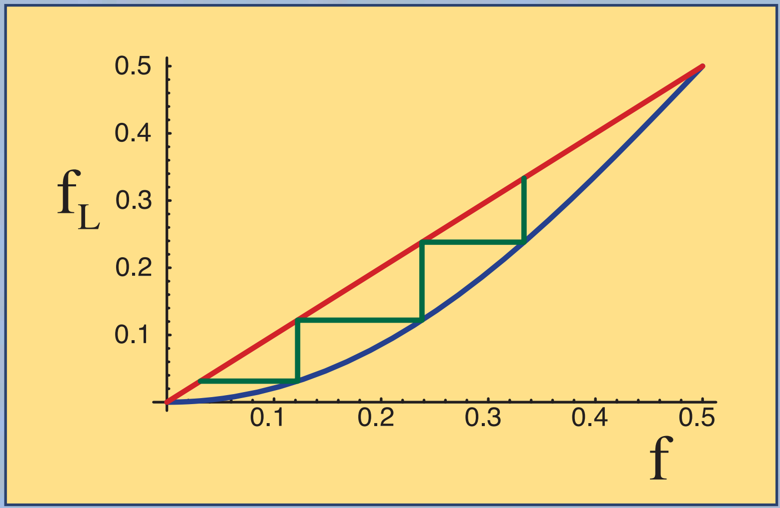
Encoded Z_{90}=ZZ_{90}



$$F_{Z^L} = f^2 + (1-f) * f^2 + f * (1-f) * F_{Z^L}$$

Encoded Failure Analysis

- Assume
  -Perfect preparation (non-deterministic) and X,Y, Z measurements
  -Perfect X_{180} , Y_{180} , Z_{180}
  -Perfect X_{ϕ} rotations
  -$(ZZ)_{90}$ and Z_{90} rotations fail with probability f on the first qubit, and conditional on success of the first, with probability f on the second. The failed qubit is Z-measured after the operation.
- Then: Encoded failure behavior = base failure behavior with failure probability $f_L \leq f^2(2-f)/(1-f(1-f))$



- Optimization required: Retry when possible, anticipated failures.

Conclusion

- **It is possible to use LOQC to simulate efficiently Quantum Computation**
- **Investigation of the threshold for specific error model**
- **Possible device for QIP, Q Communication, etc.**
- **What does this tell us about quantum computation (see R. Raussendorf, H.J. Briegel and also Nielsen)**