The Church-Turing-Deutsch Principle and its Fundamental Ramifications

Prashant *

Departement d'informatique et de recherche oprationnelle

Universite de Montreal, Quebec, Canada.

Abstract

This article is based on the idea that the Church-Turing-Deutsch (CTD) principle can play a very fundamental role in Physics similar to those of conservation laws in the construction of new physical theories. It can act as a guiding principle for new physical theories and test bed for the existing ones. This principle has already given birth to the field of Quantum Computation and can now be extended to new physical theories. We will also briefly touch upon the importance of this abstract principle based on Hilbert's idea of proving mathematical problems.

Keywords: CTD, Quantum Computation, Physics.

1 Introduction:

Of the scientific disciplines taught at today's universities, computer science is the youngest, a child born of human introspection and ingenuity. Unlike the natural sciences, which seek to discover and understand the natural world around us, computer science is a unique combination of abstraction and invention.

*Corresponding author: Prashant, e-mail: prashant@iro.umontreal.ca

"Our field is still in its embryonic stage. It's great that we haven't been around for 2000 years. We are still at a stage where very, very important results occur in front of our eyes." - Michael Rabin.

This expository paper describes what we think is one of the most interesting open problems in science. Unlike quantum gravity or high temperature superconductivity it's not a problem that has attracted a lot of public attention. Indeed, although the problem has been around for nearly twenty years, it hasn't even attracted much attention from scientists, yet its solution would be of tremendous long-term significance for both physics and computer science.

To explain the problem we need to explain an idea that is refered to as the Church-Turing-Deutsch (CTD) Principle, after the three people (Alonzo Church, Alan Turing, and David Deutsch) who contributed most to the formulation of the principle.

The CTD Principle is a descendant of a famous idea known as the Church-Turing Thesis, taught in traditional computer science courses. Just stating the CTD Principle makes it look deceptively obvious: *Every physical process can be simulated by a universal computing device.*

We do not feel surprised when told that a computer is being used to simulate a new aircraft, the explosion of a nuclear bomb, or even the creation of the Universe. The fact is, most people take it for granted that your standard desktop computer, given enough time and memory, can simulate pretty much any physical process. Some people even go the extent to imagine that Universe is a gigantic computer running under definite laws and rules. Yet our ease with the CTD Principle is an ease brought by familiarity. One hundred years ago the statement would have been far more surprising and not easy to grasp and digest.

Viewed casually, the CTD Principle still is shocking. All we have to do is look at it anew. How odd that there is a *single* physical system - albeit, an idealized system, with unbounded memory - which can be used to simulate *any other system* in the Universe! In this article we describe a little bit about this principle and its possible use as a guiding principle in future physical theories. Also I will then conjecture the existence of a Dual of CTD principle and try to formulate *in principle* assertions about physics to test its validity and limits in a formal axiomatic system way. I will discuss the history of the Principle, and try to bring forth in more detail what the Principle actually means.

2 History of the Principle:

The historical roots of the CTD Principle lie in the Church-Turing thesis, proposed by Church and Turing in the 1930s. Essentially, the Church-Turing thesis asserted that the correct way to describe a computing machine was via what we now call the *Turing machine*, which is pretty much equivalent to a modern computer, but with an unlimited memory. Remarkably, Church and Turing did their work before the advent of modern computers. Indeed, their work - done with pencil and paper - played an important role in the construction of the first universal computers, and Turing himself had a hand in the practical building and design of some of the earliest machines such as Enigma during World War 2 and other famous computing machines like ENIAC and UNIVAC.

What motivated Church and Turing to do such work? In fact the motivation was a remarkable problem posed by the great German mathematician David Hilbert. Hilbert asked whether or not there is an automatic procedure - an algorithm - that can be used, *in principle*, to solve every possible mathematical problem.

Hilbert's hope was to write down a simple cookbook recipe that would enable one, at least in principle, to determine whether a given proposition is true or false. The procedure would tell you, for example, that 1+1=2 is true, and that 7 x 12 = 83 is false. But, with patience, Hilbert's procedure would also tell you more complicated things. It would tell you, for example, that Fermat's last theorem is true, that is, there is no set of four positive numbers, a, b, c and n such that $a^n + b^n = c^n$, with $n > 2$.

In asking this question Hilbert wasn't motivated by a practical concern - such a procedure might take a very long time to establish the truth or falsity of any given proposition. But Hilbert was interested in the in principle question of whether such a procedure even exists. To Hilbert's surprise it turned out that *no such procedure exists*, and the people who proved it were Church and Turing. They proved this in two remarkable steps. First, they had to *mathematically define* what Hilbert meant by a "procedure". Remember, in Hilbert's day the notion of an algorithm or procedure was rather vague and amorphous. It certainly wasn't a well-defined mathematical concept that one could prove precise mathematical theorems about.

So Church and Turing mathematically defined a class of functions - called, unsurprisingly, the *"computable functions"* - that they claimed corresponded to the class of functions that a human mathematician would reasonably be able to compute. With this class of functions mathematically well defined, they could proceed to the second step, which was to mathematically prove that no function in their class of computable functions could ever be used to solve Hilbert's problem. Church and Turing's first step, mathematically defining the class of computable functions, put computer science on a rigorous mathematical foundation. In making such a mathematical definition, Church and Turing were essentially putting forward a thesis - now known as the Church-Turing thesis - that their mathematical class of "computable functions" correspond exactly to the class of functions that we would naturally regard as computable. The assertion of the Church-Turing thesis might be compared, for example, to Galileo and Newton's achievement in putting physics on a mathematical basis. By mathematically defining the computable functions they enabled people to reason precisely about those functions in a mathematical manner, opening up a whole new world of investigation. In mathematics, the assertion of the Church-Turing thesis might be compared to any truly fundamental definition, like that of the integers, or the class of continuous functions.

A major drawback of the Church-Turing thesis is that Church and Turing's justifications of the thesis were rather ad hoc. Turing makes a convincing case, talking at some length about the sorts of algorithmic processes that he could imagine a mathematician performing with pencil and paper, arguing that each such process could be simulated on a Turing machine. Turing's arguments were a creditable foundation for the Church-Turing thesis, but they do not rule out the possibility of finding somewhere in Nature a process that cannot be simulated on a Turing machine. If such a process were to be found, it could be used as the basis for a computing device that could compute functions not normally regarded as computable. This would force a revision of the Church-Turing thesis, although perhaps not a scrapping of the thesis entirely - what is more likely is that we would revise the definition of the Church-Turing thesis. The assumption that there are no physical i.e realistic computational devices that cannot be simulated by a Turing machine in polynomial time is often referred to as the *quantitative Church's hypothesis.* We will see that CTD principle however predicts the existence of quantum computers which may violate this assumption. Even though this may be the case it still makes sense at least for classical computers to define the following complexity classes:

Definition: *The complexity class **P** is the set of problems that can be solved in polynomial time on a universal Turing machine.*
Problems contained in P are often referred to as tractable. There are problems that although they are computable require exponential time to solve. For input sizes of practical interest these problems usually cannot be solved and are therefore called intractable. The most interesting complexity class that possibly covers a larger set of problems than P is NP.

Definition: *The complexity class **NP** is the set of problems that can be solved in polynomial time on a non-deterministic Turing machine.*
Here non-deterministic means that given the internal state and the symbol read from the tape the Turing machine has a set of possible changes in internal state, output symbol and move direction. Hence the computation is not deterministic but will follow one of many possible paths. A non-deterministic Turing machine has performed a computation when there exists at least one computation path that solves the problem. One example of an NP-problem is to list all prime numbers less than a given integer n. It is clear that P $\subseteq$ NP but one of the big open questions in computation theory is whether P is a proper

subset of NP or not. This is strongly believed to be the case but no one has been able to prove that P = NP.

3 The Church-Turing-Deutsch Principle:

Fifty years after Church and Turing's pioneering papers, in 1985 David Deutsch wrote a paper proposing a way to put the Church-Turing thesis on a firmer footing. Deutsch's idea was based on an observation that seems self-evident: *computation is inherently a physical process*, in the sense that any computation must be carried out by an actual physical computing device, and so must obey the laws of physics. Of course, just because an observation is self-evident doesn't necessarily mean that we appreciate all of its implications. Deutsch realized that this observation leads to a very interesting possibility, that of *deducing* the Church-Turing thesis from the laws of physics. This line of thought led Deutsch to propose a revision of the Church-Turing thesis, which we are calling the Church-Turing-Deutsch Principle. Just to restate it in the terms used above, the CTD Principle says that every physical process can be simulated by a universal computing device.

The way we've stated it, we haven't said what that universal computing device is. Is it a Turing machine? Is it something else? Deutsch goes further, and talks about what would be a suitable universal computing device and suggests the use of Quantum Computer as one such possible candidate. We've stated the CTD Principle in a form which is *independent of* any particular physical theory. *This means that given any physical theory we can ask whether or not the CTD Principle is true in that particular theory? That is, within the scope of that theory is it possible to construct a universal computing device that can simulate an arbitrary physical system, also within that theory?*

For Newtonian physics, the answer seems to be "yes". In 1982 Fredkin and Toffoli put forward a "billiard ball" model of computation, in which computations can be performed using colliding billiard balls - that is, using just the principles of Newtonian physics they showed that such a model could simulate conventional computers like those Turing had proposed. In turn, it is widely believed that Turing machines are sufficient to simulate all of Newtonian physics. Some detailed investigations of this question have been done by Pour-El and Richards; their work shows that Turing machines can simulate all of Newtonian physics. When we put all this reasoning together, we conclude that Newtonian physics satisfies the CTD Principle, with Fredkin and Toffoli's model as the universal computing device. But we know that Quantum Mechanics as reigned in the modern times as the most successful theory till date and we live in a world that is governed by rules that combine quantum mechanics and general relativity in some way we don't yet fully understand.

It seems to be an extremely interesting question to determine whether or not the correct ultimate physical theory obeys the CTD Principle or not. This challenge is made rather more difficult, of course, by the fact that we don't yet have such an ultimate physical theory, although several candidates have been proposed. Of course, we do have some good physical theories right now that will probably be incorporated into any hypothetical ultimate physical theory. It is a natural question whether or not such theories obey the CTD Principle. For example, one might ask whether or not it is possible to develop a model computer within quantum electrodynamics that is capable of simulating any quantum electro dynamical process. One might ask a similar question with respect to the "standard model" of particle physics, which incorporates electromagnetism as well as the strong and weak nuclear forces. We can even ask the question of whether the CTD Principle is satisfied in other more exotic theories, such as string theory or spin networks, which some people believe may serve as a basis for an ultimate physical theory.

4 The CTD Principle: A guide to the construction of new physical theories:

How likely is it that the CTD Principle is *correct*? For a physicist it is very natural to regard the CTD Principle as something that ought to be true in any reasonable physical theory. After all, the CTD Principle amounts to the belief that a limited physical system, like the human brain, ought to be capable of simulating the behavior of an arbitrary physical system, at least in principle. This is a very appealing proposition to most scientists, and certainly, to most physicists, in part because it agrees with our prejudices - we would like the world to be comprehensible - and in part because all our experience in modeling real-world phenomena suggests that it is likely correct. Albert Einstein once said:

> The most incomprehendable thing about the Universe is that it is comprehendable.

Let's grant, then, that we expect the CTD Principle to be correct in any reasonable physical theory. This may or may not be correct, but lets assume it is. *This suggests asking other way round that can the CTD Principle be used as a guideline for the construction of new physical theories?*

We can take an example in another domain where this strategy has been used: the Principle of Conservation of Energy. This Principle was originally formulated in the context of Newtonian physics, yet it survived intact through special and general relativity, quantum mechanics, and is likely to survive in future theories. Although the exact mathematical expression of the Principle has changed from theory to theory, it remains a powerful general principle governing the construction of new theories. Moreover the most interesting and intriguing aspect of Principle of conservation of energy is that it can't be proved or disproved but is a fact of Nature as we see it in every new experiment we conduct. Indeed, if a theory is proposed that does not obey some sort of conservation of energy law, physicists are unlikely to take it terribly seriously unless truly compelling reasons are given.

This is not an isolated example. Einstein credited a 1913 insight, the so-called *"Principle of Equivalence"*, as being crucial to his discovery in 1915 of the general theory of relativity. So confident was Einstein about Equivalence principle of gravitational and inertial mass that when asked about his views on Prof. Max Planck he commented thus:

> *"He was one of the finest people I have ever known and one of my best friends: but you know he did not really understand Physics."* When asked how he

> could say such a thing about Planck, Einstein said, *"during the eclipse of 1919 Planck stayed up all night to see if it would confirm the bending of light by the gravitational field of the sun. If he had really understood the way the general theory of relativity explains the equivalence of inertial and gravitational mass(Equivalence Principle), he would have gone to bed the way I did".*

There are other principles also such as the principles of relativity, gauge invariance and renormalizability, originally formulated in other context which have proved to be extremely powerful guiding principles in the development of the standard model of physics, and continue to be powerful guiding principles in the construction of new physical theories.

More recently, many physicists believe that the *holographic principle* - the idea that the best way to describe a region of space-time is through a theory defined on the region's boundary, not its interior - is a guiding principle that will help formulate an as yet unknown theory of quantum gravity. Moreover the limits on the validity of a physical theory can be judged when we check it at its boundary conditions in the extreme limiting cases. Such principles as a basis for the construction of new physical theories is not, of course, entirely scientific. Maybe the Principle of Conservation of Energy is purely a red herring, and won't show up at all in the ultimate physical theory. But constructing new physical theories is very hard work, and it's especially hard in the absence of guiding principles. So physicists are very keen to identify deep principles that they believe are likely to be correct, and which have real physical consequence. It's sort of a psychological ploy to make the construction of new physical theories possible, and it has worked extremely well in the past.

Guiding principles of this type are often rather vague, for if a principle is to be true in an as-yet-undiscovered physical theory, then of necessity it will not be possible to achieve full mathematical precision in the formulation of the principle. This is reflected in the formulation of the CTD Principle that we have seen, with many of the key terms such as *simulation* and *universal computing device* not being precisely defined. Of course, this is not to say that we do not expect instantiations of such principles in specific physical theories to be precise. The Principle of Conservation of Energy has a perfectly precise technical meaning within quantum mechanics. It has another precise technical meaning within general relativity. These technical meanings are quite different, yet both are recognizably instantiations of a more general, rather more vague, Principle of Conservation of Energy. *What would it take to regard the correctness (or otherwise) of the CTD Principle in a given physical theory as a criterion for whether that physical theory is correct or incorrect?*

It's quite possible that the answer is "not a lot". After all, so far as I'm aware no physical theory has ever been seriously proposed that is known not to satisfy the CTD Principle. On the other hand, we actually know precious little about which physical theories do (or don't) satisfy the CTD Principle: so far as we know this problem is still open for the two leading physical theories of our time, the standard model of particle physics, and general relativity. Perhaps the most extreme are people like Ed Fredkin, who have proposed that the Universe is really a giant cellular automata, and that the laws of physics should therefore be formulated as rules for such a cellular automata. This is a particularly extreme form, for it presupposes that we already know what the candidate universal computing device in the CTD Principle is.

5 Possibility of the Dual of CTD Principle:

As in Nature and many physical and natural phenomenon we find the duality of systems and concepts occuring at a very fundamental level. Even once Neils Bohr commented that the opposite of a profound truth may yet be another profound truth. This line of thinking brings us to conjecture what we call Dual of the Church-Turing-Deutsch Principle which may play an equally fundamental role in our scientific theories. So let us quote the CTD Principle given by David Deutsch[3]:

> Every ***finitely realizable physical system*** can be perfectly simulated by a ***universal model of a computing machine*** operating by finite means.

Thus we can easily see the Dual of CTD Principle which can be stated as:

> Every ***finitely realizable computing machine*** can be perfectly simulated by a ***universal model of a physical system*** operating by finite means.

Moreover we see that the fundamental theory of Nature as we know is Quantum Mechanics which was started with a brilliant discovery of quantum of energy by Prof. Planck using the concept of harmonic oscillators in his model to explain the anomalies like ultraviolate catastrophe in atomic theory. So it is tempting to conjecture that "A Simple Harmonic Oscillator in the quantal sense" which is a simplest choice of a physical model giving rise to rich structure of Quantum Mechanics can be considered as a universal model of a physical system which can simulate any computing machine operating by finite means. This motivates us to think on the lines of David Hilbert who raised some of the very fundamental questions about mathematics and its validity nearly a century ago. Hilbert was optimistic, conjecturing in a preface to his famous twenty-three unsolved problems that if a proposition can be expressed within some formal axiomatic system, then either its proof or its refutation must exist. For many years logicians followed the trail of Hilbert's conjecture. But in 1931, Kurt Godel brought down to earth logician's ambitious plans to generate "truths" with the help of computing machines. In his Incompleteness Theorem, Godel showed that for any reasonably powerful, consistent system of logic or arithmetic that we might want to describe, there will always exist statements that are true, but that cannot be proved solely on the basis of the axioms and rules of the system. Godel's argument, in sketch, is as follows. Say that in our system, we assert the following proposition: ***You cannot prove that this proposition is true.***
To see that a system containing such a proposition is either inconsistent or incomplete, consider the following. First assume that the proposition is true. Then, on the basis of what the proposition says, you can't prove that it is true, so the system must be incomplete. Now say that the proposition is false. Then on the basis of what the proposition says, you can prove that it is true, which would be inconsistent. Godel's theorem effectively defeated hopes for a complete theorem-proving system, even in the relatively simple domain of arithmetic. It did not imply, however, that automatic theorem proving was impossible – just that it is possible to give true theorems to your system that it won't be able to prove. Similarly, limits were found to the computing ability of Turing machines

and their equivalents. It was concluded that all models for abstract computation had equivalent – but not unlimited – computing power. Some problems simply cannot be solved by mechanical means. The classic example is the halting problem, the problem of asking one Turing machine to tell us if another Turing machine will ever terminate its execution on some given input. In fact, it is impossible to write a program that can tell automatically, for any other program you might hand it, if that given program will ever stop executing on a given input. In general, the halting problem "does not compute." The news was not entirely bad. It turns out that while, theoretically, there are more noncomputable problems than computable ones for computers, we are hard-pressed to come up with examples. Certainly, most of the computation humans are interested in doing lies in the computable realm. Generally, the problems that cannot be solved by formal or mechanical means are the ones that take us to metalevels of logic and computation, questions like "Can this proposition be proven true?" or "Will this program ever terminate if I run it on this input?" The inability of formal logic, separated from intuition, to answer metaquestions is intriguing in itself, one of those recurring roadblocks to mechanical computation that have a way of making us feel better about our human way of solving problems. Here I attempt to write down such possible assertions for the field of physics in general:

1. *Whether there exists an algorithmic procedure in principle to solve every* ***physical*** *problem?*

2. *Can* ***every physical system*** *in principle simulate every other physical system given enough resources?*

3. *Does there exist a* ***simplest physical system*** *(Occam's razor) which can simulate any physical system given enough resources operating by finite means/or in principle?*

These deliberations naturally lead us to think that we can do a parallel treatment of physical phenomenon as Hilbert did for mathematics by raising fundamental *"in principle"* questions to test the limits and validity of mathematical principles and models. Giving such a Hilbert like treatment to physical problems naturally leads us to speculate the existence of Godel like Incompleteness Theorem for physical models and theories. Which may once and for all settle the question about the existence or non-existence of much researched and speculated Theory of Everything or Unified Theory of Physics.

I hope this treatment of physical models would lead to more fundamental inquiries about Nature and our conception of it using physical models. It would also help us to formalize and inquire about the validity and limits of physical principles as applied to Nature and our quest of Reality per se.

6 Conclusion:

Thus we have tried to portray the fundamental importance of CTD Principle and its dual which may play a key role in physical theories. Also the need to ask *in principle* questions to ascertain the limits and validity of any physical model on the lines of Hilbert's thinking. It is quite evident that computation and physical processes are two sides of the same coin in Nature. This principle can lead to a new dimension in thinking about physical reality and man's ultimate quest of understanding the essence of existence. I hope this article has served the purpose to motivate a new thinking with regards to our conception of physical models. We can also appreciate the underlying importance, CTD principle can play as a guiding principle for future theories.

Acknowledgements: I dedicate this article to Almighty in full faith. I also acknowledge the inspiration for research from Prof. Y M Gupta and also Prof. Gilles Brassard for providing encouragement and support in completing the work.

7 Reference:

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