Entanglement Verification in QKD

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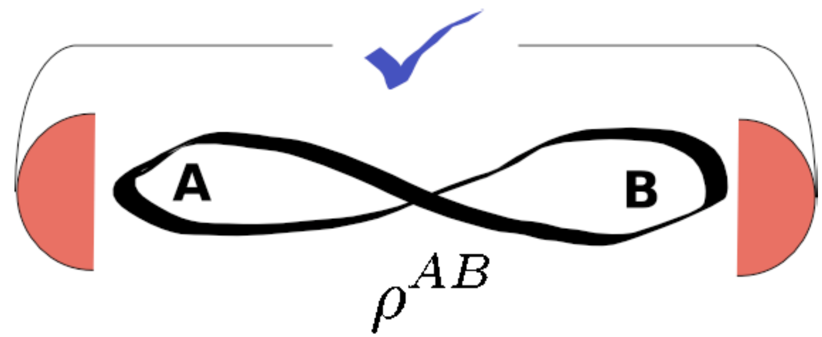
Overview

- Motivation
  - QKD and the need for entanglement
- Entanglement criteria
  - General CV entanglement
  - Qubits and modes
- Results
  - Numerical simulations
- The shared reference frame
  - The local oscillator
  - Polarisation
- Open Questions

Why entanglement verification?

Very simple picture:



- Entanglement can result in indeterministic <u>and</u> correlated measurement outcomes
- This is needed for applications like
  - Teleportation
  - Quantum Key Distribution

focus on this

A QKD protocol

authenticated

Alice **Bob**

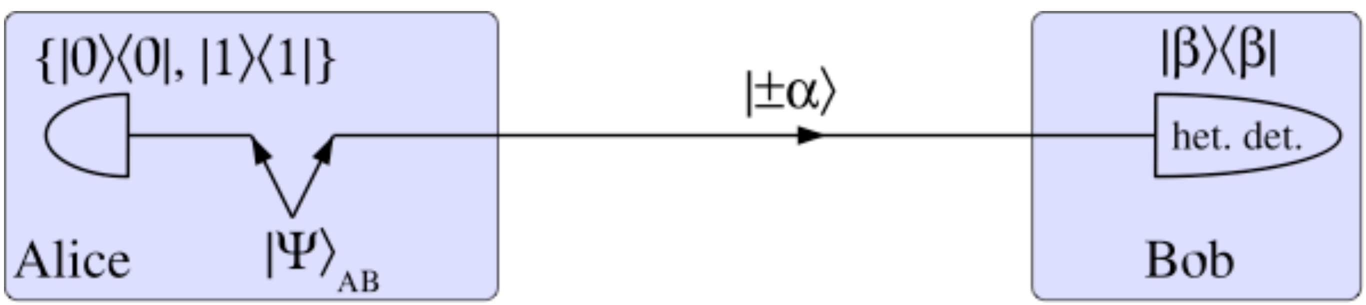
011010...

$0: \quad |-\alpha> \longrightarrow \rho_0$

$1: \quad |\alpha> \longrightarrow \rho_1$

noise loss

- The signal states must be non-orthogonal
- We need to be able to verify entanglement from the measured data



J. Rigas et al., Phys. Rev. A 73, 012341 (2006)

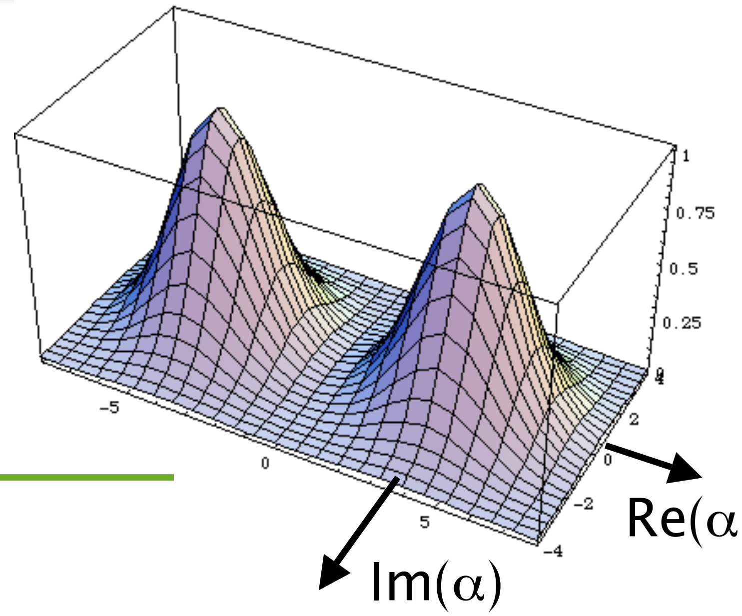
S. Lorenz et al., quant-ph/0603271

Aside: States & Measurements

Coherent states: mixture of Fock states

$$|\alpha\rangle = \mathrm{e}^{-\frac{1}{2}|\alpha|^2} \sum_{\mathrm{n}} \frac{\alpha^{\mathrm{n}}}{\sqrt{\mathrm{n}!}} |\mathrm{n}\rangle$$

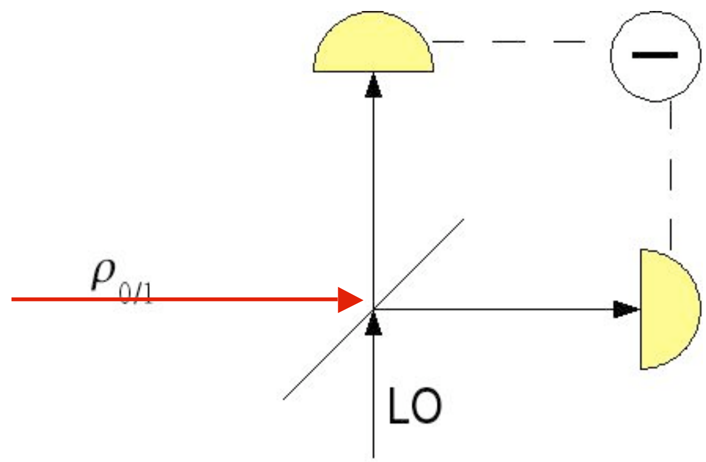
approx. laser output field



Measurements: Quadratues

$$\hat{x} = \frac{1}{\sqrt{2}}(\hat{a}^{\dagger} + \hat{a})$$ ≈Re(α)≈position of HO

$$\hat{p} = \frac{i}{\sqrt{2}}(\hat{a}^{\dagger} - \hat{a})$$ ≈Im(α) ≈momentum of HO



CV Entanglement

- Use light modes (laser light) rather than single photons for experimental convenience
- Entanglement criteria: Assume ρ^{AB} is separable:

$$\rho^{AB} = \sum_k p_k \rho_k^A \otimes \rho_k^B$$

and derive some contradiction/inequality

  - Uncertainty relation
  - Positive maps, e.g. transposition

$$(\rho^{AB})^{PT} = \sum p_k \rho_k^A \otimes (\rho_k^B)^T \geq 0$$

$$(\rho)^{PT} \ngeq 0 \Rightarrow \rho \text{ entangled}$$

Qubit-Mode Entanglement

- Construct a matrix χ containing all measured data with the property $\chi(\rho) \geq 0$
- Check the positivity of $\chi(\rho^{PT})$

$$\chi(\rho^{PT}) \ngeq 0 \Rightarrow \rho \text{ entangled}$$

J. Rigas et al., Phys. Rev. A 73, 012341 (2006)

- The construction is as follows:

$$\chi = \begin{pmatrix} \left\langle |0\rangle\langle 0| \otimes B \right\rangle & \left\langle |0\rangle\langle 1| \otimes B \right\rangle \\ \left\langle |1\rangle\langle 0| \otimes B \right\rangle & \left\langle |1\rangle\langle 1| \otimes B \right\rangle \end{pmatrix} \qquad B = \begin{pmatrix} \mathrm{id} & \hat{x} & \hat{p} \\ \hat{x} & \widehat{x^2} & \widehat{xp} \\ \hat{x} & \widehat{px} & \widehat{p^2} \end{pmatrix}$$

Partial transpose:

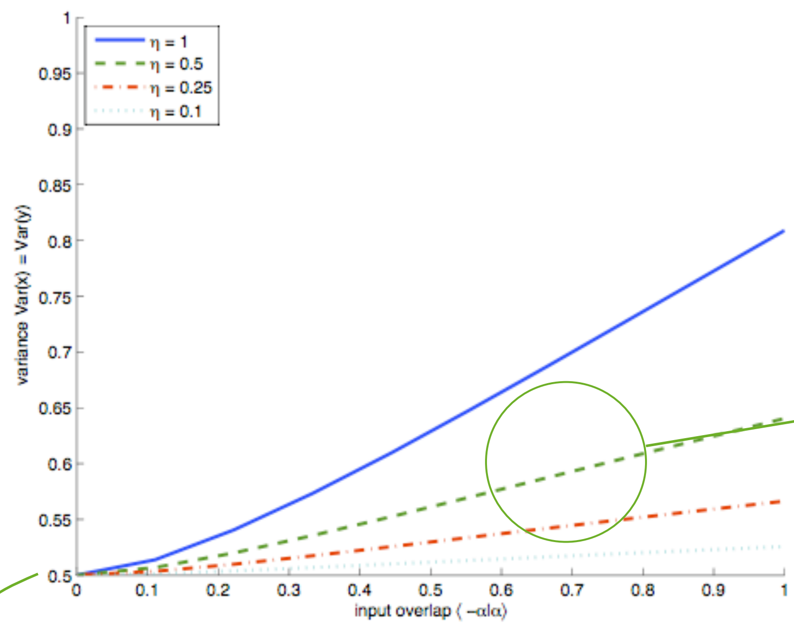
$$\mathrm{Tr}[\rho^{\mathrm{T_B}}(|0\rangle\langle 0| \otimes \hat{\mathrm{x}}\hat{\mathrm{p}})] = \mathrm{Tr}[(|0\rangle\langle 0| \otimes \hat{\mathrm{x}}\hat{\mathrm{p}})^{\mathrm{T_B}}\rho] = \mathrm{Tr}[\rho|0\rangle\langle 0| \otimes (\hat{\mathrm{x}}\hat{\mathrm{p}})^{\mathrm{T}}]$$

$$\hat{x}^T \longrightarrow \hat{x} \qquad \hat{p}^T \longrightarrow -\hat{p}$$

Results

- Observed loss: Decreased amplitudes of the signal states
- Observed noise: Broadening of the variances

$$\mathrm{Var}(\hat{\mathrm{x}}) = < \hat{\mathrm{x}}^2 >_\rho - < \hat{\mathrm{x}} >_\rho^2$$



Bigger overlap allows for more excess noise

entangled states

i.e $\chi(\rho^{\mathrm{PT}}) \not\geq 0$

from uncertainty: $\mathrm{Var}(\hat{\mathrm{x}})\mathrm{Var}(\hat{\mathrm{y}}) \geq \frac{1}{4}$

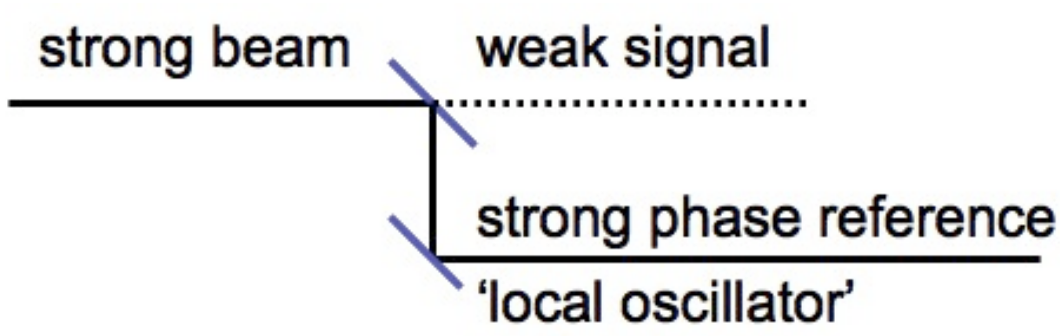
J. Rigas et al., Phys. Rev. A 73, 012341 (2006)

Including the reference frame

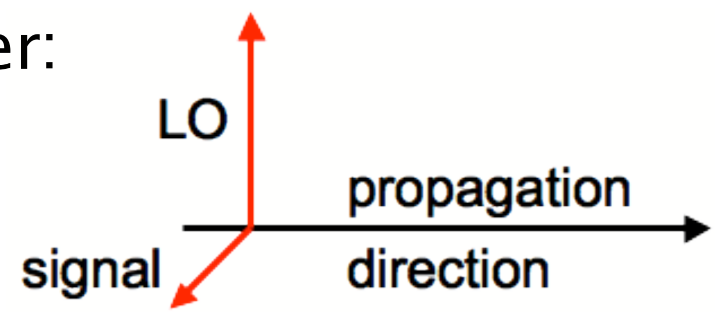
- Coherent states have a complex amplitude

$$\alpha = |\alpha| \exp i\theta$$

some phase, which
needs a reference



- The local oscillator is also transmitted
- Usually in the same fiber:



Stokes Operators

- 3 basis directions:
  - linear
  - 45° rotated
  - circular polarisation

$$
\begin{aligned}
\hat{S}_0 &= \hat{n}_H + \hat{n}_V \\
\hat{S}_1 &= \hat{n}_H - \hat{n}_V \\
\hat{S}_2 &= \hat{n}_\nearrow - \hat{n}_\searrow \\
\hat{S}_3 &= \hat{n}_R - \hat{n}_L
\end{aligned}
$$

Commutator:

$$\boxed{[\hat{S}_1, \hat{S}_2] = 2i\hat{S}_3}$$

Uncertainty:

$$\boxed{\mathrm{Var}(\hat{\mathrm{S}}_1)\mathrm{Var}(\hat{\mathrm{S}}_2) \geq |\langle \hat{\mathrm{S}}_3 \rangle|^2}$$

Polarisation Entanglement

What happens to the EVM?

$$\chi = \begin{pmatrix} \left\langle |0\rangle\langle 0| \otimes B \right\rangle & \left\langle |0\rangle\langle 1| \otimes B \right\rangle \\ \left\langle |1\rangle\langle 0| \otimes B \right\rangle & \left\langle |1\rangle\langle 1| \otimes B \right\rangle \end{pmatrix} \quad B = \begin{pmatrix} \mathrm{id} & \hat{S}_0 & \hat{S}_1 & \ldots \\ \hat{S}_0 & \ddots & & \\ \hat{S}_1 & & & \\ \vdots & & & \end{pmatrix}$$

•The same criterion holds: $\chi(\rho^{PT}) \not\geq 0 \Rightarrow \rho \text{ entangled}$

•The transposition can again be moved to the observables:

$$\hat{S}_0^T = \hat{S}_0 \qquad \hat{S}_1^T = \hat{S}_1$$
$$\hat{S}_2^T = \hat{S}_2 \qquad \hat{S}_3^T = -\hat{S}_3$$

We can again express $\chi(\rho^{PT})$ in terms of measured values

Open Questions

- Will the criterion hold for other (arbitrary) measurement operators?
- Which properties of the measurement operators are important for entanglement detection?
- Given the expectation values of a set of operators, is there a corresponding quantum states?
- Can we make the criterion necessary?

Summary

- We measure the 1st and 2nd moments of a set of non-commuting operators
- These are arranged in the Matrix χ
- χ is positive for all separable state, such that $\chi(\rho^{PT}) \ngeq 0$ is a sufficient condition for entanglement

References

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  - Measuring the Quantum State of Light, U. Leonhardt
  - Methods in Theoretical Quantum Optics, S. Barnett & P. Radmore
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  - “Introduction to the basics of entanglement theory in in continuous-variable systems”, J. Eisert & M. Plenio, quant-ph/0312071
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  - Comment on the above, A. Miranowicz & M. Piani, quant-ph/0603239
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