PLL Loop Parameters

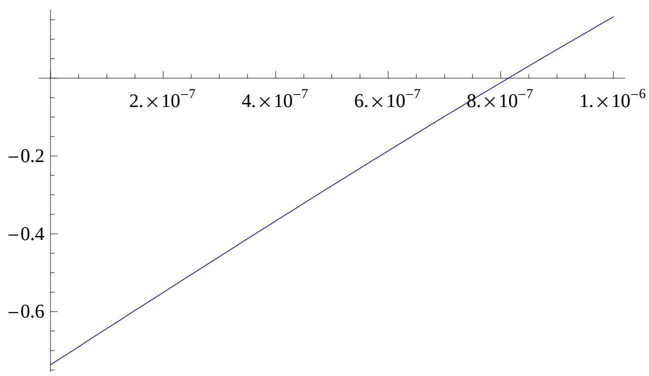
Definition of Transfer Function and Desired Loop Parameters

```
Remove["Global`*"]

f[T_] = ϕ - ArcTan[γ/(ωc T (1 + T31 + T41))] + ArcTan[ωc T] + ArcTan[ωc T T31] + ArcTan[ωc T T41];

DeanSet = {ϕ → .8343, γ → 1.115, ωc → 1.2566 * 10^5, T31 → 2.5,
   T41 → 0.4, Kϕ → 0.005, Kvco → 44 * 10^6, A → -1, n → 4882, LoopCap → 0.5079 10^-9};
AndrewSet = {ϕ → 47.8 * π/180, γ → 1.115, ωc → 2 π .5 * 10^6, T31 → 2.5, T41 → 0.4,
   Kvco → 40 * 10^6, Kϕ → .005, Kvco → 40 * 10^6, A → -1, n → 100, LoopCap → .5 10^-9};
```

Deans Settings (test)

- ### Time Constants

```
Plot[f[T1] /. DeanSet, {T1, 0, 10^-6}]
```




```
Tsoln = FindRoot[f[T] == 0 /. DeanSet, {T, 10^-8}];

T1 = T /. Tsoln;
T3 = T1 * T31 /. DeanSet;
T4 = T1 * T41 /. DeanSet;

T2 = γ/(ωc^2 (T1 + T3 + T4)) /. DeanSet;

A0 = (Kϕ * Kvco * Abs[A])/(ωc^2 n) Sqrt[(1 + ωc^2 T2^2)/((1 + ωc^2 T1^2) (1 + ωc^2 T3^2) (1 + ωc^2 T4^2))] /. DeanSet;
```

10^9 T1
10^9 T2
10^9 T3
10^9 T4
10^9 A0

813.689252548

22 251.4117332

2034.22313137

325.475701019

8.16123760776

Physical Values

$$C1 = \frac{T1}{T2} A0;$$

$$C2 = \left(1 - \frac{T1}{T2}\right) A0;$$

$$R2 = \frac{T2}{C2};$$

C4 = (LoopCap /. DeanSet);

$$C3 = C4 \frac{4\, T3\, T4}{(T3 - T4)^2};$$

$$R3 = \frac{T3 + T4}{2\,(C3 + C4)};$$

$$R4 = \frac{T3 + T4}{2\, C4};$$

Capacitors [pF]

C1 * 10^{12}
C2 * 10^{12}
C3 * 10^{12}
C4 * 10^{12}

298.440000506

7862.79760725

460.680272109

507.9

- **Resistors [Ω]**

R2
R3
R4

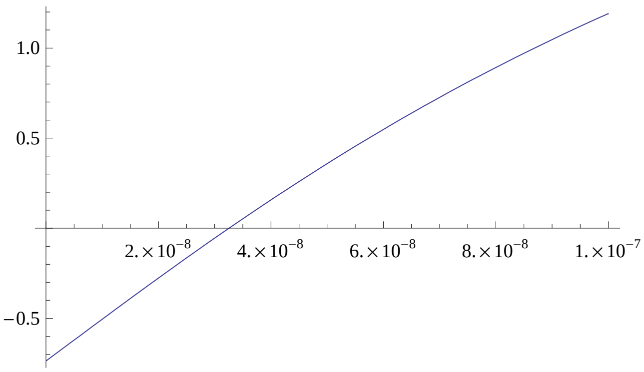
2829.96114674

1218.12249348

2322.99550343

My Settings

```
Plot[f[T1] /. AndrewSet, {T1, 10^-10, 10^-7}]
```




```
Tsoln = FindRoot[f[T] == 0 /. AndrewSet, {T, 10^-8}];
T1 = T /. Tsoln;
T3 = T1 * T31 /. AndrewSet;
T4 = T1 * T41 /. AndrewSet;
```

$$T2 = \frac{\gamma}{\omega c^2\,(T1 + T3 + T4)}\ /.\ \text{AndrewSet};$$

$$A0 = \frac{K\phi * Kvco * Abs[A]}{\omega c^2\, n}\sqrt{\frac{1 + \omega c^2\, T2^2}{\left(1 + \omega c^2\, T1^2\right)\left(1 + \omega c^2\, T3^2\right)\left(1 + \omega c^2\, T4^2\right)}}\ /.\ \text{AndrewSet};$$

10^9 **T1**
10^9 **T2**
10^9 **T3**
10^9 **T4**
10^9 **A0**

32.5481288276

889.988692097

81.3703220689

13.019251531

0.579476762672

$$C1 = \frac{T1}{T2} A0;$$

$$C2 = \left(1 - \frac{T1}{T2}\right) A0;$$

$$R2 = \frac{T2}{C2};$$

C4 = (LoopCap /. AndrewSet);

$$C3 = C4 \frac{4\, T3\, T4}{(T3 - T4)^2};$$

$$R3 = \frac{T3 + T4}{2\,(C3 + C4)};$$

$$R4 = \frac{T3 + T4}{2\, C4};$$

- **Capacitors [pF]**

$C1 * 10^{12}$
$C2 * 10^{12}$
$C3 * 10^{12}$
$C4 * 10^{12}$

21.1922741171

558.284488555

453.514739229

500.

- **Resistors [Ω]**

R2
R3
R4

1594.14905902

49.4956028033

94.3895735999

Inverted Equations In Terms of Physical Components

- **Consistency Check**

- **Time Constants**

ExpSet = $\{eC1 \to 21.19 * 10^{-12}$, $eC2 \to 558.28 * 10^{-12}$,
$eC3 \to 453.5 * 10^{-12}$, $eC4 \to 500 * 10^{-12}$, $eR2 \to 1594.15$, $eR3 \to 49.5$, $eR4 \to 94.4\}$;

$$\mathbf{eA0} = (\mathbf{eC1} + \mathbf{eC2})\ /.\ \mathbf{ExpSet};$$
$$\mathbf{eT2} = (\mathbf{eR2}\ \mathbf{eC2})\ /.\ \mathbf{ExpSet};$$
$$\mathbf{eT1} = \left(\frac{\mathbf{eC1}}{\mathbf{eA0}}\right) \mathbf{eT2}\ /.\ \mathbf{ExpSet};$$

$$\alpha = 4\left(1 + \frac{\mathbf{eC4}}{\mathbf{eC3}}\right);$$
$$\beta = \alpha\ \mathbf{eR4}\ \mathbf{eC4};$$
$$\delta = (2\ \mathbf{eR4}\ \mathbf{eC4})^2;$$
$$\frac{\beta + \sqrt{\beta^2 - \alpha\,\delta}}{\alpha}\ /.\ \mathbf{ExpSet}$$
$$\frac{\beta - \sqrt{\beta^2 - \alpha\,\delta}}{\alpha}\ /.\ \mathbf{ExpSet}$$

$8.13795745162 \times 10^{-8}$

$1.30204254838 \times 10^{-8}$

$$\mathbf{eT4} = \frac{\beta - \sqrt{\beta^2 - \alpha\,\delta}}{\alpha}\ /.\ \mathbf{ExpSet};$$
$$\mathbf{eT3} = (2\ \mathbf{eR4}\ \mathbf{eC4} - \mathbf{eT4})\ /.\ \mathbf{ExpSet};$$

■ Bandwidth and Phase Margin

$$\mathbf{Plot}\left[\omega^4\left(1 + (\omega\,\mathbf{T1})^2\right)\left(1 + (\omega\,\mathbf{T3})^2\right)\left(1 + (\omega\,\mathbf{T4})^2\right) - \left(\frac{\mathbf{K\phi\ Kvco\ A}}{\mathbf{n\ eA0}}\right)^2\left(1 + (\omega\,\mathbf{T2})^2\right)\ /.\ \mathbf{AndrewSet},\ \left\{\omega,\ 3\ 10^6,\ 3.2\ 10^6\right\}\right]$$

$$\Delta\omega = \omega\ /.\ \mathbf{FindRoot}\left[\omega^4\left(1 + (\omega\,\mathbf{T1})^2\right)\left(1 + (\omega\,\mathbf{T3})^2\right)\left(1 + (\omega\,\mathbf{T4})^2\right) - \left(\frac{\mathbf{K\phi\ Kvco\ A}}{\mathbf{n\ eA0}}\right)^2\left(1 + (\omega\,\mathbf{T2})^2\right)\ /.\ \mathbf{AndrewSet},\ \left\{\omega,\ 3 * 10^6\right\}\right]$$

$3.14162354738 \times 10^6$

$$\mathbf{e\phi} = \mathbf{ArcTan}\left[\frac{\Upsilon}{\Delta\omega\ \mathbf{eT1}\left(1 + \frac{\mathbf{eT3}}{\mathbf{eT1}} + \frac{\mathbf{eT4}}{\mathbf{eT1}}\right)}\right] - \mathbf{ArcTan}[\Delta\omega\ \mathbf{eT1}] - \mathbf{ArcTan}\left[\Delta\omega\ \mathbf{eT1}\ \frac{\mathbf{eT3}}{\mathbf{eT1}}\right] - \mathbf{ArcTan}\left[\Delta\omega\ \mathbf{eT1}\ \frac{\mathbf{eT4}}{\mathbf{eT1}}\right]\ /.\ \mathbf{AndrewSet};$$

Bandwidth [kHz]

$$\frac{\Delta\omega}{2\,\pi}\ 10^{-3}$$

500.0049169

Phase Margin [degrees]

$$\frac{180}{\pi}\ \mathbf{e\phi}$$

47.7974175337

Old Parameters

Time Constants

```
ExpSet =
  {eC1 → 1 * 10^-9, eC2 → 1 * 10^-9, eC3 → 1 * 10^-9, eC4 → 1000 * 10^-12, eR2 → 1000, eR3 → 1000, eR4 → 1000};
eA0 = (eC1 + eC2) /. ExpSet;
eT2 = (eR2 eC2) /. ExpSet;
```

$$eT1 = \left(\frac{eC1}{eA0}\right) eT2 \text{ /. ExpSet;}$$

$$\alpha = 4\left(1 + \frac{eC4}{eC3}\right);$$

$$\beta = \alpha\, eR4\, eC4;$$

$$\delta = (2\, eR4\, eC4)^2;$$

$$\frac{\beta + \sqrt{\beta^2 - \alpha\,\delta}}{\alpha} \text{ /. ExpSet}$$

$$\frac{\beta - \sqrt{\beta^2 - \alpha\,\delta}}{\alpha} \text{ /. ExpSet}$$

$$\frac{1}{8}\left(\frac{1}{125\,000} + \frac{1}{125\,000\sqrt{2}}\right)$$

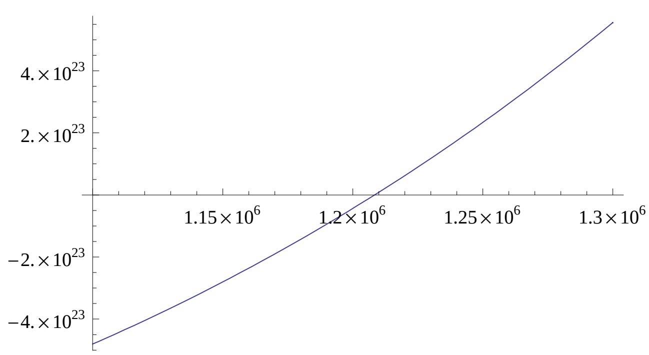
$$\frac{1}{8}\left(\frac{1}{125\,000} - \frac{1}{125\,000\sqrt{2}}\right)$$

$$eT4 = \frac{\beta - \sqrt{\beta^2 - \alpha\,\delta}}{\alpha} \text{ /. ExpSet;}$$

$$eT3 = (2\, eR4\, eC4 - eT4) \text{ /. ExpSet;}$$

Bandwidth and Phase Margin

$$\text{Plot}\left[\omega^4 \left(1 + (\omega\, T1)^2\right)\left(1 + (\omega\, T3)^2\right)\left(1 + (\omega\, T4)^2\right) - \left(\frac{K\phi\, Kvco\, A}{n\, eA0}\right)^2 \left(1 + (\omega\, T2)^2\right) \text{ /. AndrewSet},\ \{\omega,\ 1.1\, 10^6,\ 1.3\, 10^6\}\right]$$



```
Δω = ω /. FindRoot[
    ω^4 (1 + (ω T1)^2) (1 + (ω T3)^2) (1 + (ω T4)^2) - (Kϕ Kvco A / (n eA0))^2 (1 + (ω T2)^2) /. AndrewSet, {ω, 1.2*10^6}]
```

$1.20837459798 \times 10^{6}$

```
eϕ = ArcTan[γ / (Δω eT1 (1 + eT3/eT1 + eT4/eT1))] -
    ArcTan[Δω eT1] - ArcTan[Δω eT1 eT3/eT1] - ArcTan[Δω eT1 eT4/eT1] /. AndrewSet;
```

Bandwidth [kHz]

$$\frac{\Delta\omega}{2\pi} 10^{-3}$$

192.318790376

Phase Margin [degrees]

$$\frac{180}{\pi} e\phi$$

-94.5085081737

```
N[eT3/eT1]
N[eT4/eT1]
```

3.41421356237

0.585786437627

▪ New Parameters

▪ Time Constants

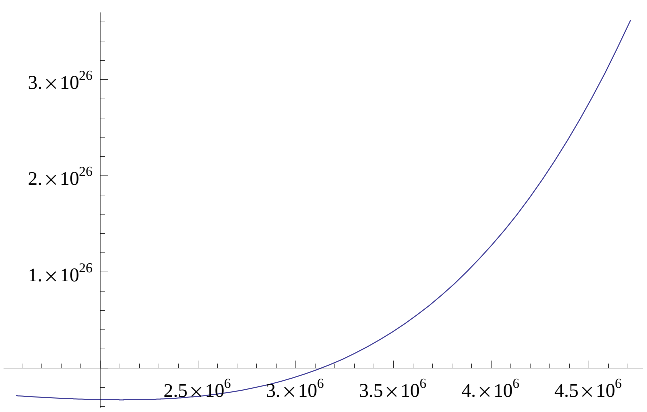
```
ExpSet = {eC1 → 22*10^-12, eC2 → 560*10^-12,
    eC3 → 470*10^-12, eC4 → 470*10^-12, eR2 → 1500, eR3 → 47, eR4 → 100};
```

```
eA0 = (eC1 + eC2) /. ExpSet;
eT2 = (eR2 eC2) /. ExpSet;
eT1 = (eC1/eA0) eT2 /. ExpSet;
```

$$\alpha = 4\left(1 + \frac{eC4}{eC3}\right);$$
$$\beta = \alpha\, eR4\, eC4;$$
$$\delta = (2\, eR4\, eC4)^2;$$
$$\frac{\beta + \sqrt{\beta^2 - \alpha\,\delta}}{\alpha} \text{ /. ExpSet}$$
$$\frac{\beta - \sqrt{\beta^2 - \alpha\,\delta}}{\alpha} \text{ /. ExpSet}$$
$$\frac{1}{8}\left(\frac{47}{125\,000\,000} + \frac{47}{125\,000\,000\,\sqrt{2}}\right)$$
$$\frac{1}{8}\left(\frac{47}{125\,000\,000} - \frac{47}{125\,000\,000\,\sqrt{2}}\right)$$
$$eT4 = \frac{\beta - \sqrt{\beta^2 - \alpha\,\delta}}{\alpha} \text{ /. ExpSet;}$$
$$eT3 = (2\, eR4\, eC4 - eT4) \text{ /. ExpSet;}$$

- **Find Bandwidth and Phase Margin:**

$$\text{Plot}\left[\omega^4\left(1 + (\omega\, T1)^2\right)\left(1 + (\omega\, T3)^2\right)\left(1 + (\omega\, T4)^2\right) - \left(\frac{K\phi\, Kvco\, A}{n\, eA0}\right)^2\left(1 + (\omega\, T2)^2\right) \text{ /. AndrewSet}, \left\{\omega, \frac{\pi}{2}10^6, \frac{3\pi}{2}10^6\right\}\right]$$



$$\Delta\omega = \omega \text{ /. FindRoot}\left[\omega^4\left(1 + (\omega\, T1)^2\right)\left(1 + (\omega\, T3)^2\right)\left(1 + (\omega\, T4)^2\right) - \left(\frac{K\phi\, Kvco\, A}{n\, eA0}\right)^2\left(1 + (\omega\, T2)^2\right) \text{ /. AndrewSet}, \left\{\omega, 3.2 * 10^6\right\}\right];$$

$$e\phi = \text{ArcTan}\left[\frac{\gamma}{\Delta\omega\, eT1\left(1 + \frac{eT3}{eT1} + \frac{eT4}{eT1}\right)}\right] - \text{ArcTan}[\Delta\omega\, eT1] - \text{ArcTan}\left[\Delta\omega\, eT1\,\frac{eT3}{eT1}\right] - \text{ArcTan}\left[\Delta\omega\, eT1\,\frac{eT4}{eT1}\right] \text{ /. AndrewSet;}$$

- **Bandwidth [kHz] (want 500 kHz)**

$$\frac{\Delta\omega}{2\,\pi}\,10^{-3}$$

```
498.172969599
```

- **Phase Margin [degrees] (want 47.8^o)**

$$\frac{180}{\pi}\,e\phi$$

```
48.314815057
```

- **Pole Ratio 3 to 1 (Want 2.5)**

$$N\left[\frac{eT3}{eT1}\right]$$

```
2.52685058943
```

- **Pole Ratio 4 to 1 (want 0.4)**

$$N\left[\frac{eT4}{eT1}\right]$$

```
0.433539020964
```

- **Capacitors C_1 to C_4 [pF]**

$$eC1*10^{12}\ /.\ ExpSet$$
$$eC2*10^{12}\ /.\ ExpSet$$
$$eC3*10^{12}\ /.\ ExpSet$$
$$eC4*10^{12}\ /.\ ExpSet$$

```
22

560

470

470
```

- **Resistors R_2 to R_3 [Ω]**

```
eR2 /. ExpSet
eR3 /. ExpSet
eR4 /. ExpSet
```

```
1500

47

100
```